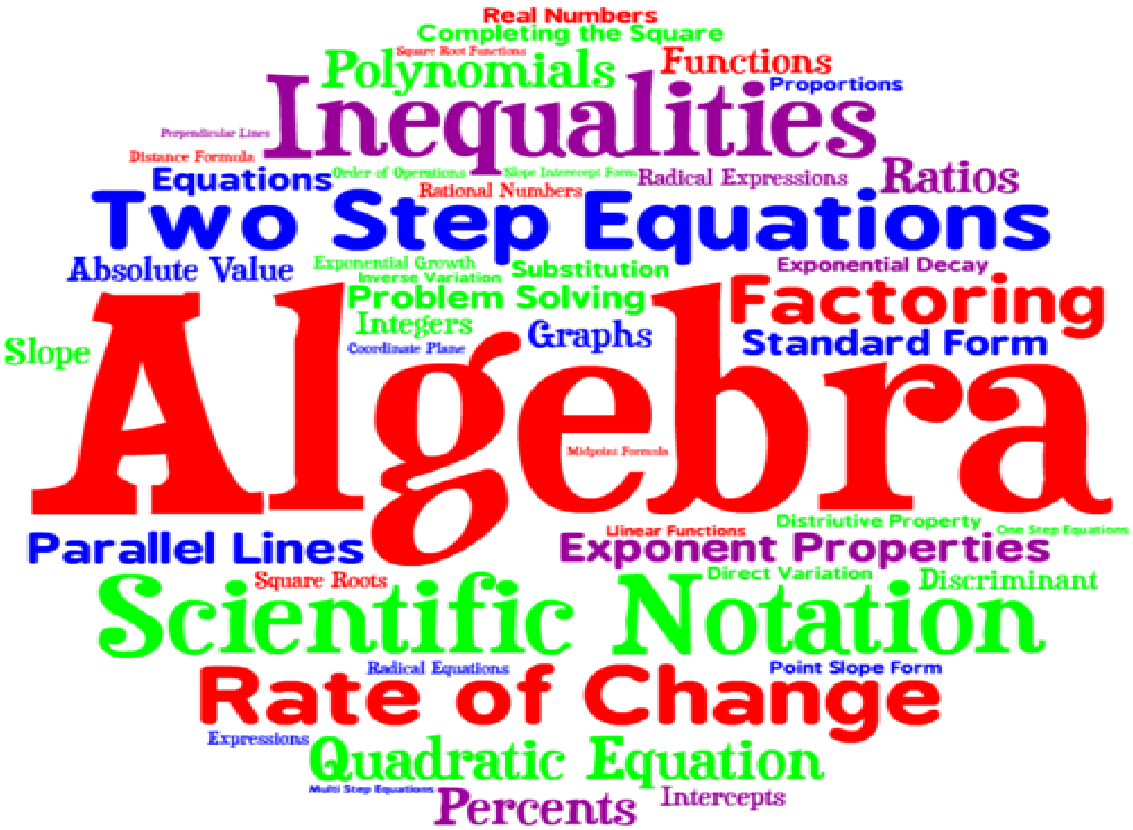
DELAWARE TECH | Academic Challenge

Level I Math Assessment Packet

Complete all **odd numbered problems** in Exercise Sets R.1 (pages 9-11), R.2 (pages 19-22), and R.3 (pages 28-31). Answers can be found in the back of the packet.

Please bring worked out problems to the first day of class on August 18, **there will be a quiz**.



Math Session Schedule*

<u>Monday, July 13, 20, 27 — 5:30 pm-7:00 pm</u>

Students who scored lower than 82% on the math assessment are required to attend July 13, 20, & 27 math sessions for instruction on content.

<u>August 3, 10 — 5:30 pm-7:00 pm</u>

August 3 & 10 sessions will be review sessions for the quiz on August 18th.

Location: Zoom on-line meeting. Link available on our web site: www.academicchallengetheflyingcow.com/new-students

Instructor: Mrs. Ingerski, hingersk@dtcc.edu

**All students are welcome to attend any/all sessions*

R.1

PART 1 OPERATIONS
The Set of Real Numbers

OBJECTIVES

a Use roster notation and set-builder notation to name sets, and distinguish among various kinds of real numbers.

b Determine which of two real numbers is greater and indicate which, using < and >; given an inequality like $a < b$, write another inequality with the same meaning; and determine whether an inequality like $-2 \le 3$ or $4 > 5$ is true.

c Graph inequalities on the number line.

d Find the absolute value of a real number.

To the student:

At the front of the text, you will find a Student Organizer card. This pullout card will help you keep track of important dates and useful contact information. You can also use it to plan time for class, study, work, and relaxation. By managing your time wisely, you will provide yourself the best possible opportunity to be successful in this course.

Find the opposite of each number.

1. 9

2. -6

3. 0

Answers

1. -9 **2.** 6 **3.** 0

a Set Notation and the Set of Real Numbers

A **set** is a collection of objects. In mathematics, we usually consider sets of numbers. The set we consider most in algebra is **the set of real numbers.** There is a real number for every point on the real-number line. Some commonly used sets of numbers are **subsets** of, or sets contained within, the set of real numbers. We begin by examining some subsets of the set of real numbers.

The set containing the numbers -5, 0, and 3 can be named $\{-5, 0, 3\}$. This method of describing sets is known as the **roster method.** We use the roster method to describe three frequently used subsets of real numbers. Note that three dots are used to indicate that the pattern continues without end.

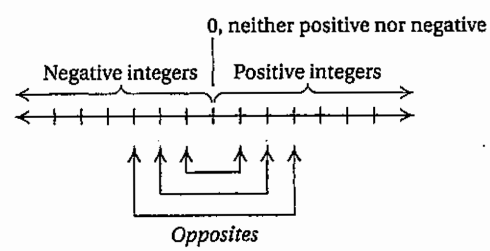
Natural numbers are those numbers used for counting: $\{1, 2, 3, \ldots\}$.

Whole numbers are the set of natural numbers with 0 included: $\{0, 1, 2, 3, \ldots\}$.

Integers are the set of whole numbers and their opposites:

$$\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}.$$

The integers can be illustrated on the real-number line as follows.



The set of integers extends infinitely to the left and to the right of 0. The **opposite** of a number is found by reflecting it across the number 0. Thus the opposite of 3 is -3. The opposite of -4 is 4. The opposite of 0 is 0. We read a symbol like -3 as either "the opposite of 3" or "negative 3."

The natural numbers are called **positive integers**. The opposites of the natural numbers (those to the left of 0) are called **negative integers**. Zero is neither positive nor negative.

Do Exercises 1–3 (in the margin at left).

Each point on the number line corresponds to a real number. In order to fill in the remaining numbers on the number line, we must describe two other subsets of the real numbers. And to do that, we need another type of set notation.

Set-builder notation is used to specify conditions under which a number is in a set. For example, the set of all odd natural numbers less than 9 can be described as follows:

$$\{x \mid x \text{ is an odd natural number less than } 9\}.$$

The set of ←
all numbers x ←
such that ←
x is an odd natural number less than 9. ←

We can easily write another name for this set using roster notation, as follows:

$\{1, 3, 5, 7\}$.

EXAMPLE 1 Name the set consisting of the first six even whole numbers using both roster notation and set-builder notation.

Roster notation: $\{0, 2, 4, 6, 8, 10\}$

Set-builder notation: $\{x \mid x \text{ is one of the first six even whole numbers}\}$

Do Exercise 4.

4. Name the set consisting of the first seven odd whole numbers using both roster notation and set-builder notation.

The advantage of set-builder notation is that we can use it to describe very large sets that may be difficult to describe using roster notation. Such is the case when we try to name the set of **rational numbers**. Rational numbers can be named using fraction notation. The following are examples of rational numbers:

$$\frac{5}{8},\ \frac{12}{-7},\ \frac{-17}{15},\ -\frac{9}{7},\ \frac{39}{1},\ \frac{0}{6}.$$

We can now describe the set of rational numbers.

RATIONAL NUMBERS

A **rational number** can be expressed as an integer divided by a nonzero integer. The set of rational numbers is

$$\left\{\frac{p}{q} \,\middle|\, p \text{ is an integer, } q \text{ is an integer, and } q \neq 0\right\}.$$

Rational numbers are numbers whose decimal representation either terminates or has a repeating block of digits.

Each of the following is a rational number:

$$\frac{5}{8} = \underbrace{0.625}_{\text{Terminating}} \quad \text{and} \quad \frac{6}{11} = \underbrace{0.545454\ldots}_{\text{Repeating}} = 0.\overline{54}.$$

The bar in $0.\overline{54}$ indicates the repeating block of digits in decimal notation.

Note that $\frac{39}{1} = 39$. Thus the set of rational numbers contains the integers.

Do Exercises 5 and 6.

Convert each fraction to decimal notation by long division and determine whether it is terminating or repeating.

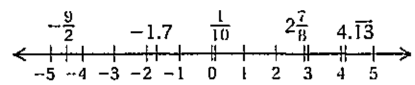
5. $\frac{11}{16}$

6. $\frac{14}{13}$

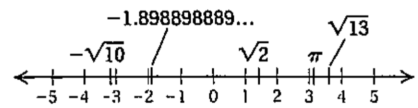
Answers

4. $\{1, 3, 5, 7, 9, 11, 13\}$; $\{x \mid x \text{ is one of the first seven odd whole numbers}\}$ 5. 0.6875; terminating 6. $1.\overline{076923}$; repeating

The real-number line has a point for every rational number.

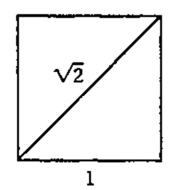


However, there are many points on the line for which there is no rational number. These points correspond to what are called **irrational numbers.**



Numbers like π, $\sqrt{2}$, $-\sqrt{10}$, $\sqrt{13}$, and $-1.898898889\ldots$ are examples of irrational numbers. The decimal notation for an irrational number *neither* terminates *nor* repeats. Recall that decimal notation for rational numbers either terminates or has a repeating block of digits.

Irrational numbers are numbers whose decimal representation neither terminates nor has a repeating block of digits. They cannot be represented as the quotient of two integers.



The irrational number $\sqrt{2}$ (read "the square root of 2") is the length of the diagonal of a square with sides of length 1. It is also the number that, when multiplied by itself, gives 2. No rational number can be multiplied by itself to get 2, although some approximations come close:

1.4 is an *approximation* of $\sqrt{2}$ because
$(1.4)^2 = (1.4)(1.4) = 1.96$;

1.41 is a better approximation because
$(1.41)^2 = (1.41)(1.41) = 1.9881$;

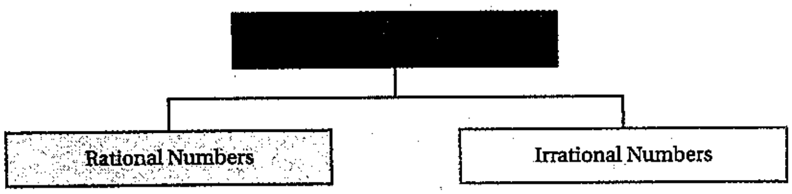
1.4142 is an even better approximation because
$(1.4142)^2 = (1.4142)(1.4142) = 1.99996164$.

We say that 1.4142 is a rational approximation of $\sqrt{2}$ because

$$(1.4142)^2 = 1.99996164 \approx 2.$$

The symbol $\approx$ means "is approximately equal to." We can find rational approximations for square roots and other irrational numbers using a calculator.

The set of all rational numbers, combined with the set of all irrational numbers, gives us the set of **real numbers**.

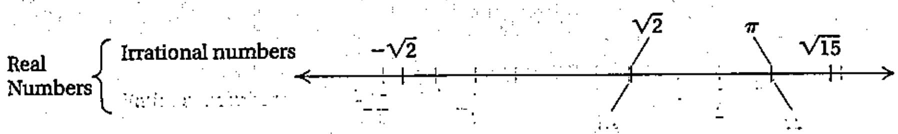


REAL NUMBERS

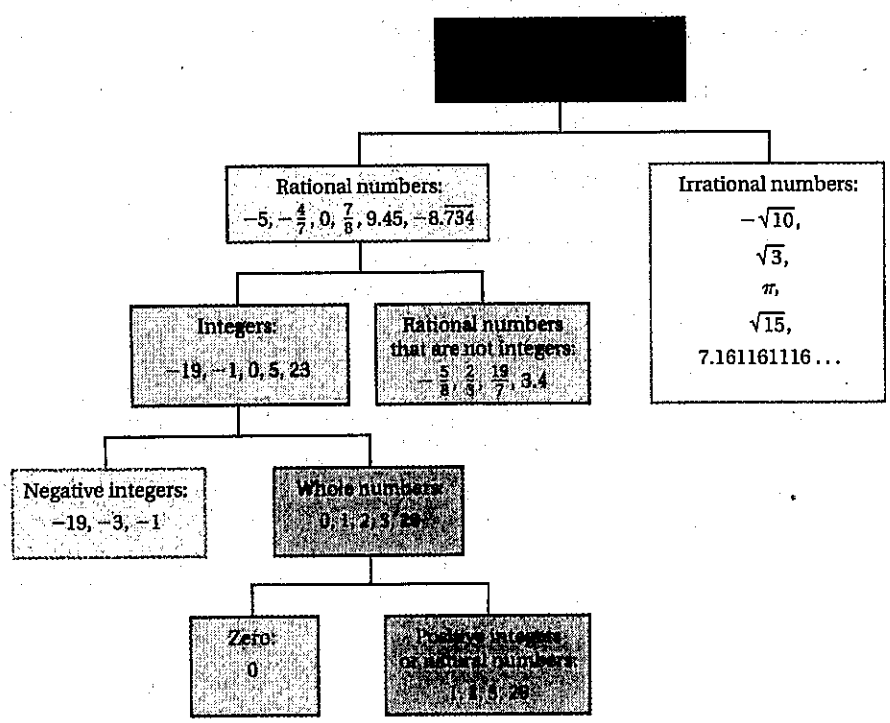
The set of **real numbers** is

$\{x \mid x \text{ is a rational number } or\ x \text{ is an irrational number}\}$.

Every point on the number line represents some real number and every real number is represented by some point on the number line.



The following figure shows the relationships among various kinds of real numbers.



Do Exercise 7.

7. Given the numbers

$20,\ -10,\ -5.34,\ 18.999,$

$\frac{11}{45},\ \sqrt{7},\ -\sqrt{2},\ \sqrt{16},\ 0,\ -\frac{2}{3},$

$9.34334333433334\ldots$:

a) Name the natural numbers.

b) Name the whole numbers.

c) Name the integers.

d) Name the irrational numbers.

e) Name the rational numbers.

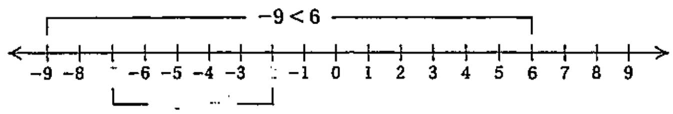
f) Name the real numbers.

Answer

7. (a) $20, \sqrt{16}$; (b) $20, \sqrt{16}, 0$; (c) $20, -10, \sqrt{16}, 0$; (d) $\sqrt{7}, -\sqrt{2}$, $9.34334333433334\ldots$; (e) $20, -10, -5.34, 18.999, \frac{11}{45}, \sqrt{16}, 0, -\frac{2}{3}$; (f) $20, -10, -5.34, 18.999, \frac{11}{45}, \sqrt{7}, -\sqrt{2}, \sqrt{16}, 0, -\frac{2}{3}$, $9.34334333433334\ldots$

b Order for the Real Numbers

Real numbers are named in order on the number line, with larger numbers named further to the right. For any two numbers on the line, the one to the left is less than the one to the right.



We use the symbol < to mean "**is less than.**" The sentence $-9 < 6$ means "−9 is less than 6." The symbol > means "**is greater than.**" The sentence $-2 > -7$ means "−2 is greater than −7." A handy mental device is to think of > or < as an arrowhead that points to the smaller number.

EXAMPLES Use either < or > for □ to write a true sentence.

2. 4 □ 9 Since 4 is to the left of 9, 4 is less than 9, so $4 < 9$.

3. −8 □ 3 Since −8 is to the left of 3, we have $-8 < 3$.

4. 7 □ −12 Since 7 is to the right of −12, then $7 > -12$.

5. −21 □ −5 Since −21 is to the left of −5, we have $-21 < -5$.

6. $-2.7 \square -\frac{3}{2}$ Since $-\frac{3}{2} = -1.5$ and −2.7 is to the left of −1.5, we have $-2.7 < -\frac{3}{2}$.

7. $1\frac{1}{4} \square -2.7$ The answer is $1\frac{1}{4} > -2.7$.

8. 4.79 □ 4.97 The answer is $4.79 < 4.97$.

9. −8.45 □ 1.32 The answer is $-8.45 < 1.32$.

10. $\frac{5}{8} \square \frac{7}{11}$ We convert to decimal notation $\left(\frac{5}{8} = 0.625 \text{ and } \frac{7}{11} = 0.6363\ldots\right)$ and compare. Thus, $\frac{5}{8} < \frac{7}{11}$.

Do Exercises 8–16.

Insert < or > for □ to write a true sentence.

8. −5 □ −4

9. $-\frac{1}{4} \square -\frac{1}{2}$

10. 87 □ 67

11. $-9.8 \square -4\frac{2}{3}$

12. 6.78 □ −6.77

13. $-\frac{4}{5} \square -0.86$

14. $\frac{14}{29} \square \frac{17}{32}$

15. $-\frac{12}{13} \square -\frac{14}{15}$

16. 1.8 □ 1.08

All positive real numbers are greater than zero and all negative real numbers are less than zero.

> If x is a positive real number, then $x > 0$.
> If x is a negative real number, then $x < 0$.

Note that $-8 < 5$ and $5 > -8$ are both true. These are **inequalities.** Every true inequality yields another true inequality if we interchange the numbers or variables and reverse the direction of the inequality sign.

> $a < b$ also has the meaning $b > a$.

Answers

8. < 9. > 10. > 11. < 12. >
13. > 14. < 15. > 16. >

EXAMPLES Write a different inequality with the same meaning.

11. $a < -5$ The inequality $-5 > a$ has the same meaning.

12. $-3 > -8$ The inequality $-8 < -3$ has the same meaning.

Do Exercises 17 and 18.

Write a different inequality with the same meaning.

17. $x > 6$

18. $-4 < 7$

Expressions like $a \le b$ and $b \ge a$ are also **inequalities**. We read $a \le b$ as "**a is less than or equal to b.**" We read $a \ge b$ as "**a is greater than or equal to b.**" If a is nonnegative, then $a \ge 0$.

EXAMPLES Write true or false.

13. $-8 \le 5.7$ True since $-8 < 5.7$ is true.

14. $-8 \le -8$ True since $-8 = -8$ is true.

15. $-7 \ge 4\frac{1}{3}$ False since neither $-7 > 4\frac{1}{3}$ nor $-7 = 4\frac{1}{3}$ is true.

16. $-\frac{2}{3} \ge -\frac{5}{4}$ True since $-\frac{2}{3} = -0.666\ldots$ and $-\frac{5}{4} = -1.25$ and $-0.666\ldots > -1.25$.

Do Exercises 19–22.

Write true or false.

19. $6 \ge -9.4$

20. $-18 \le -18$

21. $-7.6 \le -10\frac{4}{5}$

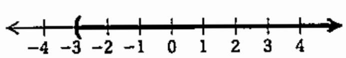
22. $-\frac{24}{27} \ge -\frac{25}{28}$

(c) Graphing Inequalities on the Number Line

Some replacements for the variable in an inequality make it true and some make it false. A replacement that makes it true is called a **solution**. The set of all solutions is called the **solution set**. A **graph** of an inequality is a drawing that represents its solution set.

EXAMPLE 17 Graph the inequality $x > -3$ on the number line.

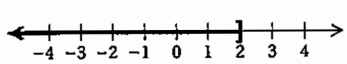
The solutions consist of all real numbers greater than -3, so we shade all numbers greater than -3. Note that -3 is not a solution. We indicate this by using a parenthesis at -3.



The graph represents the solution set $\{x \mid x > -3\}$. Numbers in this set include $-2.6, -1, 0, \pi, \sqrt{2}, 3\frac{7}{8}, 5$, and 123.

EXAMPLE 18 Graph the inequality $x \le 2$ on the number line.

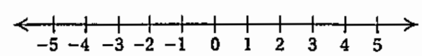
We make a drawing that represents the solution set $\{x \mid x \le 2\}$. The graph consists of 2 as well as the numbers less than 2. We shade all numbers to the left of 2 and use a bracket at 2 to indicate that it is also a solution.



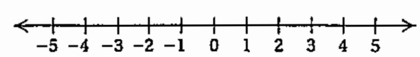
Do Exercises 23–28. (Exercises 25–28 are on the following page.)

Graph each inequality.

23. $x > -1$

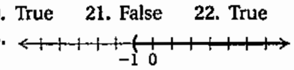


24. $x \le 5$

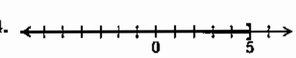


Answers

17. $6 < x$ 18. $7 > -4$ 19. True
20. True 21. False 22. True
23.



24.

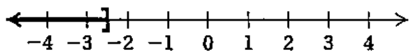


Match each inequality with one of the graphs shown below.

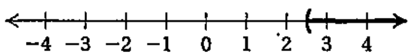
25. $x \le -\frac{5}{2}$ **26.** $x > 0$

27. $-4 > x$ **28.** $2 \le x$

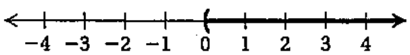
a)



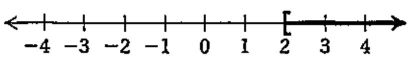
b)



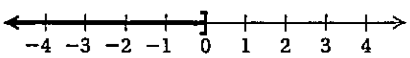
c)



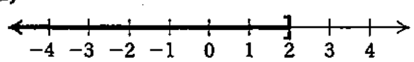
d)



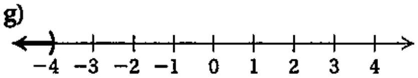
e)



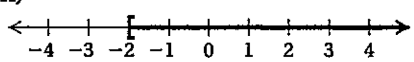
f)



g)



h)



Find the absolute value.

29. $\left|-\frac{1}{4}\right|$ **30.** $|2|$

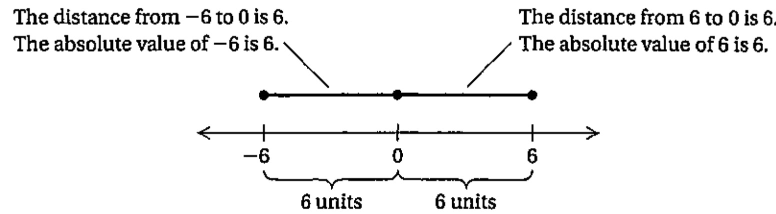
31. $\left|\frac{3}{2}\right|$ **32.** $|-2.3|$

Answers

25. (a) 26. (c) 27. (g) 28. (d)
29. $\frac{1}{4}$ 30. 2 31. $\frac{3}{2}$ 32. 2.3

(d) Absolute Value

We see that numbers like −6 and 6 are the same distance from 0 on the number line. We call the distance of a number from 0 on the number line the **absolute value** of the number. Since distance is always a nonnegative number, the absolute value of a number is always greater than or equal to 0.



The **absolute value** of a number is its distance from zero on the number line. We use the symbol $|x|$ to represent the absolute value of a number x.

To find absolute value:

1. If a number is negative, its absolute value is its opposite.
2. If a number is positive or zero, its absolute value is the same as the number.

EXAMPLES Find the absolute value.

19. $|-7|$ The distance of −7 from 0 is 7, so $|-7|$ is 7.

20. $|12|$ The distance of 12 from 0 is 12, so $|12|$ is 12.

21. $|0|$ The distance of 0 from 0 is 0, so $|0|$ is 0.

22. $\left|\frac{4}{5}\right| = \frac{4}{5}$

23. $|-3.86| = 3.86$

Do Exercises 29–32.

*A more formal definition of $|x|$ is given in Section R.2.

R.1 Exercise Set

For Extra Help MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

a Given the numbers $-6, 0, 1, -\frac{1}{2}, -4, \frac{7}{9}, 12, -\frac{6}{5}, 3.45, 5\frac{1}{2}, \sqrt{3}, \sqrt{25}, -\frac{12}{3}, 0.131331333133331\ldots$:

1. Name the natural numbers.

2. Name the whole numbers.

3. Name the rational numbers.

4. Name the integers.

5. Name the real numbers.

6. Name the irrational numbers.

Given the numbers $-\sqrt{5}, -3.43, -11, 12, 0, \frac{11}{34}, -\frac{7}{13}, \pi, -3.565665666566665\ldots$:

7. Name the whole numbers.

8. Name the natural numbers.

9. Name the integers.

10. Name the rational numbers.

11. Name the irrational numbers.

12. Name the real numbers.

Use roster notation to name each set.

13. The set of all letters in the word "math"

14. The set of all letters in the word "solve"

15. The set of all positive integers less than 13

16. The set of all odd whole numbers less than 13

17. The set of all even natural numbers

18. The set of all negative integers greater than -4

Use set-builder notation to name each set.

19. $\{0, 1, 2, 3, 4, 5\}$

20. $\{4, 5, 6, 7, 8, 9, 10\}$

21. The set of all rational numbers

22. The set of all real numbers

23. The set of all real numbers greater than -3

24. The set of all real numbers less than or equal to 21

b Use either $<$ or $>$ for $\square$ to write a true sentence.

25. $13 \square 0$

26. $18 \square 0$

27. $-8 \square 2$

28. $7 \square -7$

29. $-8 \square 8$

30. $0 \square -11$

31. $-8 \square -3$

32. $-6 \square -3$

33. $-2 \square -12$

34. $-7 \square -10$

35. $-9.9 \square -2.2$

36. $-13\frac{1}{5} \square \frac{11}{250}$

37. $37\frac{1}{5} \square -1\frac{67}{100}$

38. $-13.99 \square -8.45$

39. $\frac{6}{13} \square \frac{13}{25}$

40. $-\frac{14}{15} \square -\frac{27}{53}$

Write a different inequality with the same meaning.

41. $-8 > x$

42. $x < 7$

43. $-12.7 \leq y$

44. $10\frac{2}{3} \geq t$

Write true or false.

45. $6 \leq -6$

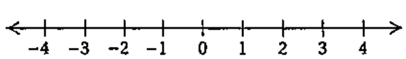
46. $-7 \leq -7$

47. $5 \geq -8.4$

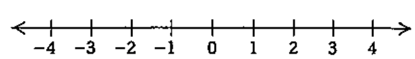
48. $-11 \geq -13\frac{1}{2}$

c Graph each inequality.

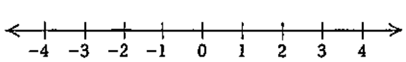
49. $x < -2$



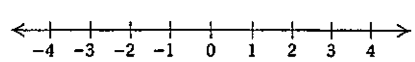
50. $x < -1$



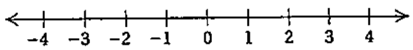
51. $x \leq -2$



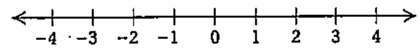
52. $x \geq -1$



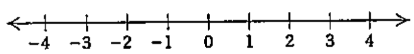
53. $x > -3.3$



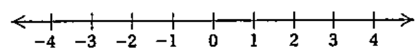
54. $x < 0$



55. $x \geq 2$



56. $x \leq 0$



d Find the absolute value.

57. $|-6|$ **58.** $|-3|$ **59.** $|28|$ **60.** $|16|$ **61.** $|-35|$

62. $|-127|$ **63.** $\left|-\frac{2}{3}\right|$ **64.** $\left|-\frac{13}{8}\right|$ **65.** $|42.8|$ **66.** $|16.4|$

67. $|986|$ **68.** $|465|$ **69.** $\left|\frac{0}{-7}\right|$ **70.** $\left|\frac{0}{-15}\right|$

Synthesis

To the student and the instructor: The Synthesis exercises found at the end of every exercise set challenge students to combine concepts or skills studied in that section or in preceding parts of the text.

Use either $\leq$ or $\geq$ for $\square$ to write a true sentence.

71. $|-3| \square 5$ **72.** $|-5| \square |-2|$ **73.** $|4| \square |-7|$ **74.** $|-8| \square |8|$

75. List the following numbers in order from least to greatest.

$$\frac{1}{11},\ 1.1\%,\ \frac{2}{7},\ 0.3\%,\ 0.11,\ \frac{1}{8}\%,\ 0.009,\ \frac{99}{1000},\ 0.286,\ \frac{1}{8},\ 1\%,\ \frac{9}{100}$$

R.2 Operations with Real Numbers

OBJECTIVES

- **a** Add real numbers.
- **b** Find the opposite, or additive inverse, of a number.
- **c** Subtract real numbers.
- **d** Multiply real numbers.
- **e** Divide real numbers.

We now review addition, subtraction, multiplication, and division of real numbers.

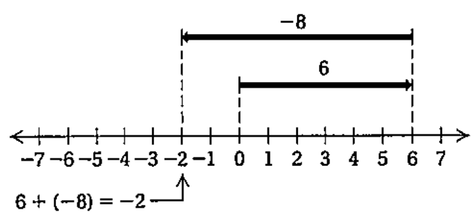
a Addition

To gain an understanding of addition of real numbers, we first add using the number line.

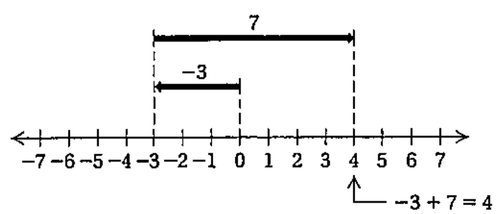
> **ADDITION ON THE NUMBER LINE**
>
> To find $a + b$, we start at 0, move to a, and then move according to b.
>
> - If b is positive, move to the right.
> - If b is negative, move to the left.
> - If b is 0, stay at a.

EXAMPLES

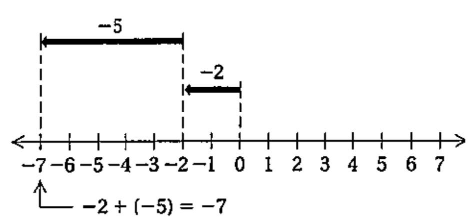
1. $6 + (-8) = -2$: We begin at 0 and move 6 units right since 6 is positive. Then we move 8 units left since -8 is negative. The answer is -2.



2. $-3 + 7 = 4$: We begin at 0 and move 3 units left since -3 is negative. Then we move 7 units right since 7 is positive. The answer is 4.



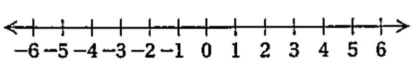
3. $-2 + (-5) = -7$: We begin at 0 and move 2 units left since -2 is negative. Then we move 5 units further left since -5 is negative. The answer is -7.



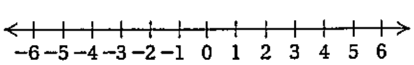
Do Exercises 1–4.

Add using the number line.

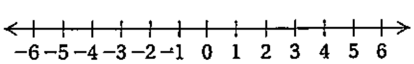
1. $-5 + 9$



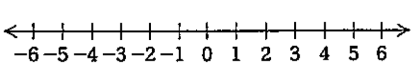
2. $4 + (-2)$



3. $3 + (-8)$



4. $-5 + 5$



Answers

1. 4 2. 2 3. −5 4. 0

You may have noticed some patterns in the preceding examples. These lead us to rules for adding without using the number line.

RULES FOR ADDITION OF REAL NUMBERS

1. *Positive numbers*: Add the numbers. The result is positive.
2. *Negative numbers*: Add absolute values. Make the answer negative.
3. *A positive and a negative number*:
   - If the numbers have the same absolute value, the answer is 0.
   - If the numbers have different absolute values, subtract the smaller absolute value from the larger. Then:
     a) If the positive number has the greater absolute value, make the answer positive.
     b) If the negative number has the greater absolute value, make the answer negative.
4. *One number is zero*: The sum is the other number.

Rule 4 is known as the **identity property of 0.** It says that for any real number a, $a + 0 = a$.

EXAMPLES Add without using the number line.

4. $-13 + (-8) = -21$ Two negatives. Add the absolute values: $|-13| + |-8| = 13 + 8 = 21$. Make the answer *negative*: -21.

5. $-2.1 + 8.5 = 6.4$ One negative, one positive. Find the absolute values: $|-2.1| = 2.1$; $|8.5| = 8.5$. Subtract the smaller absolute value from the larger: $8.5 - 2.1 = 6.4$. The *positive* number, 8.5, has the larger absolute value, so the answer is *positive*, 6.4.

6. $-48 + 31 = -17$ One negative, one positive. Find the absolute values: $|-48| = 48$; $|31| = 31$. Subtract the smaller absolute value from the larger: $48 - 31 = 17$. The *negative* number, -48, has the larger absolute value, so the answer is *negative*, -17.

7. $2.6 + (-2.6) = 0$ One positive, one negative. The numbers have the same absolute value. The sum is 0.

8. $-\frac{5}{9} + 0 = -\frac{5}{9}$ One number is zero. The sum is $-\frac{5}{9}$.

9. $-\frac{3}{4} + \frac{9}{4} = \frac{6}{4} = \frac{3}{2}$

10. $-\frac{2}{3} + \frac{5}{8} = -\frac{16}{24} + \frac{15}{24} = -\frac{1}{24}$

Do Exercises 5–14.

Add.

5. $-7 + (-11)$

6. $-8.9 + (-9.7)$

7. $-\frac{6}{5} + \left(-\frac{23}{5}\right)$

8. $-\frac{3}{10} + \left(-\frac{2}{5}\right)$

9. $-7 + 7$

10. $-7.4 + 0$

11. $4 + (-7)$

12. $-7.8 + 4.5$

13. $\frac{3}{8} + \left(-\frac{5}{8}\right)$

14. $-\frac{3}{5} + \frac{7}{10}$

Answers

5. -18 6. -18.6 7. $-\frac{29}{5}$ 8. $-\frac{7}{10}$ 9. 0 10. -7.4 11. -3 12. -3.3 13. $-\frac{1}{4}$ 14. $\frac{1}{10}$

Find the opposite, or additive inverse, of each number.

15. -14

16. $\frac{2}{3}$

17. 0

Caution!

A symbol such as -8 is usually read "negative 8." It could be read "the opposite of 8," because the opposite of 8 is -8. It could also be read "the additive inverse of 8," because the additive inverse of 8 is -8. When a variable is involved, as in a symbol like $-x$, it can be read "the opposite of x" or "the additive inverse of x" but *not* "negative x," because we do not know whether the symbol represents a positive number, a negative number, or 0. It is never correct to read -8 as "minus 8."

18. Evaluate $-a$ when $a = 9$.

19. Evaluate $-a$ when $a = -\frac{3}{5}$.

20. Evaluate $-(-a)$ when $a = -5.9$.

21. Evaluate $-(-a)$ when $a = \frac{2}{3}$.

(b) Opposites, or Additive Inverses

Suppose we add two numbers that are **opposites**, such as 4 and -4. The result is 0. When opposites are added, the result is always 0. Such numbers are also called **additive inverses**. Every real number has an opposite, or additive inverse.

[illegible]

Two numbers whose sum is 0 are called **opposites**, or **additive inverses**, of each other.

EXAMPLES Find the opposite, or additive inverse, of each number.

11. 8.6 The opposite of 8.6 is -8.6 because $8.6 + (-8.6) = 0$.

12. 0 The opposite of 0 is 0 because $0 + 0 = 0$.

13. $-\frac{7}{9}$ The opposite of $-\frac{7}{9}$ is $\frac{7}{9}$ because $-\frac{7}{9} + \frac{7}{9} = 0$.

To name the opposite, or additive inverse, we use the symbol $-$, and read the symbolism $-a$ as "the opposite of a" or "the additive inverse of a."

Do Exercises 15–17.

EXAMPLE 14 Evaluate $-x$ and $-(-x)$ when (a) $x = 23$ and (b) $x = -5$.

a) If $x = 23$, then $-x = -23 = -23$. The opposite, or additive inverse, of 23 is -23.

If $x = 23$, then $-(-x) = -(-23) = 23$. The opposite of the opposite of 23 is 23.

b) If $x = -5$, then $-x = -(-5) = 5$.
If $x = -5$, then $-(-x) = -(-(-5)) = -(5) = -5$.

Note in Example 14(b) that an extra set of parentheses is used to show that we are substituting the negative number -5 for x. Symbolism like $--x$ is not considered meaningful.

Do Exercises 18–21.

We can use the symbolism $-a$ for the opposite of a to restate the definition of opposite.

OPPOSITES, OR ADDITIVE INVERSES

For any real number a, the **opposite**, or **additive inverse**, of a, which is $-a$, is such that

$$a + (-a) = (-a) + a = 0.$$

Answers

15. 14 16. $-\frac{2}{3}$ 17. 0 18. -9
19. $\frac{3}{5}$ 20. -5.9 21. $\frac{2}{3}$

Signs of Numbers

A negative number is sometimes said to have a "negative sign." A positive number is said to have a "positive sign." When we replace a number with its opposite, or additive inverse, we can say that we have "changed its sign."

EXAMPLES Change the sign. (Find the opposite, or additive inverse.)

15. -3 $\quad -(-3) = 3$

16. $-\frac{3}{8}$ $\quad -\left(-\frac{3}{8}\right) = \frac{3}{8}$

17. 0 $\quad -0 = 0$

18. 14 $\quad -(14) = -14$

Do Exercise 22.

We can now use the concept of opposite to give a more formal definition of absolute value.

For any real number a, the **absolute value** of a, denoted $|a|$, is given by

$$|a| = \begin{cases} a, & \text{if } a \geq 0, \quad \text{For example, } |8| = 8 \text{ and } |0| = 0. \\ -a, & \text{if } a < 0. \quad \text{For example, } |-5| = -(-5) = 5. \end{cases}$$

(The absolute value of a is a if a is nonnegative. The absolute value of a is the opposite of a if a is negative.)

c Subtraction

The difference $a - b$ is the unique number c for which $a = b + c$. That is, $a - b = c$ if c is the number such that $a = b + c$.

For example, $3 - 5 = -2$ because $3 = 5 + (-2)$. That is, -2 is the number that when added to 5 gives 3. Although this illustrates the formal definition of subtraction, we generally use the following when we subtract.

For any real numbers a and b,

$$a - b = a + (-b).$$

(We can subtract by adding the opposite (additive inverse) of the number being subtracted.)

EXAMPLES Subtract.

19. $3 - 5 = 3 + (-5) = -2$ — Changing the sign of 5 and adding

20. $7 - (-3) = 7 + (3) = 10$ — Changing the sign of -3 and adding

21. $-19.4 - 5.6 = -19.4 + (-5.6) = -25$

22. $-\frac{4}{3} - \left(-\frac{2}{5}\right) = -\frac{4}{3} + \frac{2}{5} = -\frac{20}{15} + \frac{6}{15} = -\frac{14}{15}$

22. Change the sign.

a) 11

b) -17

c) 0

d) x

e) $-x$

Subtract.

23. $8 - (-9)$

24. $-10 - 6$

25. $5 - 8$

26. $-23.7 - 5.9$

27. $-2 - (-5)$

28. $-\frac{11}{12} - \left(-\frac{23}{12}\right)$

29. $\frac{2}{3} - \left(-\frac{5}{6}\right)$

30. a) $17 - 23$

b) $-17 - 23$

c) $-17 - (-23)$

Answers

22. (a) -11; (b) 17; (c) 0; (d) $-x$; (e) x 23. 17 24. -16 25. -3 26. -29.6 27. 3 28. 1 29. $\frac{3}{2}$ 30. (a) -6; (b) -40; (c) 6

Do Exercises 23–30 on the preceding page.

d Multiplication

We know how to multiply positive numbers. What happens when we multiply a positive number and a negative number?

Do Exercise 31.

[illegible]

To multiply a positive number and a negative number, multiply their absolute values. Then make the answer negative.

EXAMPLES Multiply.

23. $-3 \cdot 5 = -15$

24. $6 \cdot (-7) = -42$

25. $(-1.2)(4.5) = -5.4$

26. $3 \cdot \left(-\frac{1}{2}\right) = \frac{3}{1} \cdot \left(-\frac{1}{2}\right) = -\frac{3}{2}$

Note in Example 25 that the parentheses indicate multiplication.

Do Exercises 32–34.

What happens when we multiply two negative numbers?

Do Exercise 35.

THE PRODUCT OF TWO NEGATIVE NUMBERS

To multiply two negative numbers, multiply their absolute values. The answer is positive.

EXAMPLES Multiply.

27. $-3 \cdot (-5) = 15$

28. $-5.2(-10) = 52$

29. $(-8.8)(-3.5) = 30.8$

30. $\left(-\frac{3}{4}\right) \cdot \left(-\frac{5}{2}\right) = \frac{15}{8}$

Do Exercises 36–38.

e Division

DIVISION

The quotient $a \div b$, or $\frac{a}{b}$, where $b \neq 0$, is that unique real number c for which $a = b \cdot c$.

The definition of division parallels the one for subtraction. Using this definition and the rules for multiplying, we can see how to handle signs when dividing.

31. Look for a pattern and complete.

$4 \cdot 5 = 20$	$-2 \cdot 5 =$
$3 \cdot 5 = 15$	$-3 \cdot 5 =$
$2 \cdot 5 =$	$-4 \cdot 5 =$
$1 \cdot 5 =$	$-5 \cdot 5 =$
$0 \cdot 5 =$	$-6 \cdot 5 =$
$-1 \cdot 5 =$	

Multiply.

32. $-4 \cdot 6$

33. $(3.5)(-8.1)$

34. $-\frac{4}{5} \cdot 10$

35. Look for a pattern and complete.

$4(-5) = -20$	$-1(-5) =$
$3(-5) = -15$	$-2(-5) =$
$2(-5) =$	$-3(-5) =$
$1(-5) =$	$-4(-5) =$
$0(-5) =$	$-5(-5) =$

Multiply.

36. $-8(-9)$

37. $\left(-\frac{4}{5}\right) \cdot \left(-\frac{2}{3}\right)$

38. $(-4.7)(-9.1)$

Answers

31. 10, 5, 0, −5, −10, −15, −20, −25, −30
32. −24 33. −28.35 34. −8 35. −10, −5, 0, 5, 10, 15, 20, 25 36. 72 37. $\frac{8}{15}$
38. 42.77

EXAMPLES Divide.

31. $\frac{10}{-2} = -5$, because $-5 \cdot (-2) = 10$

32. $\frac{-32}{4} = -8$, because $-8 \cdot (4) = -32$

33. $\frac{-25}{-5} = 5$, because $5 \cdot (-5) = -25$

34. $\frac{40}{-4} = -10$

35. $-10 \div 5 = -2$

36. $\frac{-10}{-40} = \frac{1}{4}$, or 0.25

37. $\frac{-10}{-3} = \frac{10}{3}$, or $3.\overline{3}$

The rules for division and multiplication are the same.

> To multiply or divide two real numbers:
>
> 1. Multiply or divide the absolute values.
> 2. If the signs are the same, then the answer is positive.
> 3. If the signs are different, then the answer is negative.

Do Exercises 39–42.

Divide.

39. $\frac{-28}{-14}$

40. $125 \div (-5)$

41. $\frac{-75}{25}$

42. $-4.2 \div (-21)$

Excluding Division by Zero

We cannot divide a nonzero number n by zero. By the definition of division, $n/0$ would be some number that when multiplied by 0 gives n. But when any number is multiplied by 0, the result is 0. Thus the only possibility for n would be 0.

Consider $0/0$. We might say that it is 5 because $5 \cdot 0 = 0$. We might also say that it is -8 because $-8 \cdot 0 = 0$. In fact, $0/0$ could be any number at all. So, division by 0 does not make sense. Division by 0 is not defined and is not possible.

EXAMPLES Divide, if possible.

38. $\frac{7}{0}$ Not defined: Division by 0.

39. $\frac{0}{7} = 0$ The quotient is 0 because $0 \cdot 7 = 0$.

40. $\frac{4}{x - x}$ Not defined: $x - x = 0$ for any x.

Do Exercises 43–46.

Divide, if possible.

43. $\frac{8}{0}$

44. $\frac{0}{9}$

45. $\frac{17}{2x - 2x}$

46. $\frac{3x - 3x}{x - x}$

Division and Reciprocals

Two numbers whose product is 1 are called **reciprocals** (or **multiplicative inverses**) of each other.

> [illegible]
>
> Every nonzero real number a has a **reciprocal** (or **multiplicative inverse**) $1/a$. The reciprocal of a positive number is positive. The reciprocal of a negative number is negative.

Find the reciprocal of each number.

47. $\frac{3}{8}$

48. $-\frac{4}{5}$

49. 18

50. -4.3

51. 0.5

Answers

39. 2 40. -25 41. -3 42. 0.2
43. Not defined 44. 0 45. Not defined
46. Not defined 47. $\frac{8}{3}$ 48. $-\frac{5}{4}$ 49. $\frac{1}{18}$
50. $-\frac{1}{4.3}$, or $-\frac{10}{43}$ 51. $\frac{1}{0.5}$, or 2

52. Complete the following table.

[illegible]	[illegible]	[illegible]
$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{3}{2}$
$\frac{4}{9}$		
$-\frac{3}{4}$		
0.25		
8		
−5		
0		

EXAMPLES Find the reciprocal of each number.

41. $\frac{4}{5}$ The reciprocal is $\frac{5}{4}$, because $\frac{4}{5} \cdot \frac{5}{4} = 1$.

42. 8 The reciprocal is $\frac{1}{8}$, because $8 \cdot \frac{1}{8} = 1$.

43. $-\frac{2}{3}$ The reciprocal is $-\frac{3}{2}$, because $-\frac{2}{3} \cdot \left(-\frac{3}{2}\right) = 1$.

44. 0.25 The reciprocal is $\frac{1}{0.25}$ or 4, because $0.25 \cdot 4 = 1$.

Remember that a number and its reciprocal (multiplicative inverse) have the same sign. Do *not* change the sign when taking the reciprocal of a number. On the other hand, when finding an opposite (additive inverse), change the sign.

Do Exercises 47–52. (Exercises 47–51 are on the preceding page.)

We know that we can subtract by adding an opposite, or additive inverse. Similarly, we can divide by multiplying by a reciprocal.

RECIPROCALS AND DIVISION

For any real numbers a and b, $b \neq 0$,

$$a \div b = \frac{a}{b} = a \cdot \frac{1}{b}.$$

(To divide, we can multiply by the reciprocal of the divisor.)

We sometimes say that we "invert the divisor and multiply."

EXAMPLES Divide by multiplying by the reciprocal of the divisor.

45. $\frac{1}{4} \div \quad = \frac{1}{4} \cdot \quad = \frac{5}{12}$ "Inverting" the divisor, $\frac{3}{5}$, and multiplying

46. $\frac{2}{3} \div \left(\quad\right) = \frac{2}{3} \cdot \left(\quad\right) = -\frac{18}{12}$, or $-\frac{3}{2}$

47. $-\frac{5}{7} \div \quad = -\frac{5}{7} \cdot \quad = -\frac{5}{21}$

Do Exercises 53–56.

Divide by multiplying by the reciprocal of the divisor.

53. $-\frac{3}{4} \div \frac{7}{8}$

54. $-\frac{12}{5} \div \left(-\frac{7}{15}\right)$

55. $-\frac{3}{8} \div (-5)$

56. $\frac{4}{5} \div \left(-\frac{1}{10}\right)$

The following properties can be used to make sign changes.

SIGN CHANGES IN FRACTION NOTATION

For any numbers a and b, $b \neq 0$,

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b} \quad \text{and} \quad \frac{-a}{-b} = \frac{a}{b}.$$

We can illustrate this property with $a = 4$ and $b = 9$:

$$\frac{-4}{9} = \frac{4}{-9} = -\frac{4}{9} \quad \text{and} \quad \frac{-4}{-9} = \frac{4}{9}.$$

Answers

52. $-\frac{4}{9}, \frac{9}{4}; \frac{3}{4}, -\frac{4}{3}; -0.25, \frac{1}{0.25}$, or 4; $-8, \frac{1}{8}$; 5, $-\frac{1}{5}$; 0, does not exist 53. $-\frac{6}{7}$ 54. $\frac{36}{7}$ 55. $\frac{3}{40}$ 56. −8

R.2 Exercise Set

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a Add.

1. $-10 + (-18)$
2. $-13 + (-12)$
3. $7 + (-2)$
4. $7 + (-5)$
5. $-8 + (-8)$
6. $-6 + (-6)$
7. $7 + (-11)$
8. $9 + (-12)$
9. $-16 + 6$
10. $-21 + 11$
11. $-26 + 0$
12. $0 + (-32)$
13. $-8.4 + 9.6$
14. $-6.3 + 8.2$
15. $-2.62 + (-6.24)$
16. $-2.73 + (-8.46)$
17. $-\frac{5}{9} + \frac{2}{9}$
18. $-\frac{3}{7} + \frac{1}{7}$
19. $-\frac{11}{12} + \left(-\frac{5}{12}\right)$
20. $-\frac{3}{8} + \left(-\frac{7}{8}\right)$
21. $\frac{2}{5} + \left(-\frac{3}{10}\right)$
22. $-\frac{3}{4} + \frac{1}{8}$
23. $-\frac{2}{5} + \frac{3}{4}$
24. $-\frac{5}{6} + \left(-\frac{7}{8}\right)$

b Evaluate $-a$ for each of the following.

25. $a = -4$
26. $a = -9$
27. $a = 3.7$
28. $a = 0$

Find the opposite (additive inverse).

29. 10
30. $-\frac{2}{3}$
31. 0
32. $-2x$

c Subtract.

33. $3 - 7$
34. $8 - 13$
35. $-5 - 9$
36. $-6 - 14$
37. $23 - 23$
38. $23 - (-23)$
39. $-23 - 23$
40. $-23 - (-23)$
41. $-6 - (-11)$
42. $-7 - (-12)$
43. $10 - (-5)$
44. $28 - (-16)$

45. $15.8 - 27.4$

46. $17.2 - 34.9$

47. $-18.01 - 11.24$

48. $-19.04 - 15.76$

49. $-\frac{21}{4} - \left(-\frac{7}{4}\right)$

50. $-\frac{16}{5} - \left(-\frac{3}{5}\right)$

51. $-\frac{1}{3} - \left(-\frac{1}{12}\right)$

52. $-\frac{7}{8} - \left(-\frac{5}{2}\right)$

53. $-\frac{3}{4} - \frac{5}{6}$

54. $-\frac{2}{3} - \frac{4}{5}$

55. $\frac{1}{3} - \frac{4}{5}$

56. $-\frac{4}{7} - \left(-\frac{5}{9}\right)$

d Multiply.

57. $3(-7)$

58. $5(-8)$

59. $-2 \cdot 4$

60. $-5 \cdot 9$

61. $-8(-3)$

62. $-5(-7)$

63. $-7 \cdot 16$

64. $-8 \cdot 19$

65. $-6(-5.7)$

66. $-7(-6.1)$

67. $-\frac{3}{5} \cdot \frac{4}{7}$

68. $-\frac{5}{4} \cdot \frac{11}{3}$

69. $-3\left(-\frac{2}{3}\right)$

70. $-5\left(-\frac{3}{5}\right)$

71. $-3(-4)(5)$

72. $-6(-8)(9)$

73. $(-4.2)(-6.3)$

74. $(-7.4)(-9.6)$

75. $-\frac{9}{11} \cdot \left(-\frac{11}{9}\right)$

76. $-\frac{13}{7} \cdot \left(-\frac{5}{2}\right)$

77. $-\frac{2}{3} \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right)$

78. $-\frac{4}{5} \cdot \left(-\frac{4}{5}\right) \cdot \left(-\frac{4}{5}\right)$

e Divide, if possible.

79. $\frac{-8}{4}$ **80.** $\frac{-16}{2}$ **81.** $\frac{56}{-8}$ **82.** $\frac{63}{-7}$ **83.** $-77 \div (-11)$

84. $-48 \div (-6)$ **85.** $\frac{-5.4}{-18}$ **86.** $\frac{-8.4}{-12}$ **87.** $\frac{5}{0}$ **88.** $\frac{92}{0}$

89. $\frac{0}{32}$ **90.** $\frac{0}{17}$ **91.** $\frac{9}{y - y}$ **92.** $\frac{2x - 2x}{2x - 2x}$

Find the reciprocal of each number.

93. $\frac{3}{4}$ **94.** $\frac{9}{10}$ **95.** $-\frac{7}{8}$ **96.** $-\frac{5}{6}$ **97.** 25

98. -65 **99.** 0.2 **100.** 0.8 **101.** $-\frac{a}{b}$ **102.** $\frac{1}{8x}$

Divide.

103. $\frac{2}{7} \div \left(-\frac{11}{3}\right)$ **104.** $\frac{3}{5} \div \left(-\frac{6}{7}\right)$ **105.** $-\frac{10}{3} \div \left(-\frac{2}{15}\right)$ **106.** $-\frac{12}{5} \div \left(-\frac{3}{10}\right)$

107. $18.6 \div (-3.1)$ **108.** $39.9 \div (-13.3)$ **109.** $(-75.5) \div (-15.1)$ **110.** $(-12.1) \div (-0.11)$

111. $-48 \div 0.4$ **112.** $520 \div (-0.13)$ **113.** $\frac{3}{4} \div \left(-\frac{2}{3}\right)$ **114.** $\frac{5}{8} \div \left(-\frac{1}{2}\right)$

115. $-\frac{5}{4} \div \left(-\frac{3}{4}\right)$ **116.** $-\frac{5}{9} \div \left(-\frac{5}{6}\right)$ **117.** $-\frac{2}{3} \div \left(-\frac{4}{9}\right)$ **118.** $-\frac{3}{5} \div \left(-\frac{5}{8}\right)$

119. $-\frac{3}{8} \div \left(-\frac{8}{3}\right)$ **120.** $-\frac{5}{8} \div \left(-\frac{5}{6}\right)$ **121.** $-6.6 \div 3.3$ **122.** $-44.1 \div (-6.3)$

123. $\frac{-12}{-13}$

124. $\frac{-1.9}{20}$

125. $\frac{48.6}{-30}$

126. $\frac{-17.8}{3.2}$

127. $\frac{-9}{17 - 17}$

128. $\frac{-8}{-6 + 6}$

129. Complete the following table.

NUMBER	OPPOSITE (Additive Inverse)	RECIPROCAL (Multiplicative Inverse)
$\frac{2}{3}$		
$-\frac{5}{4}$		
0		
1		
−4.5		
$x, x \neq 0$		

130. Complete the following table.

NUMBER	OPPOSITE (Additive Inverse)	RECIPROCAL (Multiplicative Inverse)
$-\frac{3}{8}$		
$\frac{7}{10}$		
−1		
0		
−6.4		
$a, a \neq 0$		

Skill Maintenance

This heading indicates that the exercises that follow are *Skill Maintenance exercises,* which review any skill previously studied in the text. You can expect such exercises in every exercise set. Answers to *all* skill maintenance exercises are found at the back of the book. If you miss an exercise, restudy the objective shown in red.

Given the numbers $\sqrt{3}$, −12.47, −13, 26, π, 0, $-\frac{23}{32}$, $\frac{7}{11}$, 4.57557555755557...: [R.1a]

131. Name the whole numbers.

132. Name the natural numbers.

133. Name the integers.

134. Name the irrational numbers.

135. Name the rational numbers.

136. Name the real numbers.

Use either < or > for □ to write a true sentence. [R.1b]

137. −7 □ 8

138. 5 □ $\frac{3}{8}$

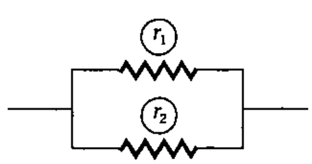
139. −45.6 □ −23.8

140. 123 □ −10

Synthesis

141. The reciprocal of an electric resistance is called *conductance.* When two resistors are connected in parallel, the conductance is the sum of the conductances,

$$\frac{1}{r_1} + \frac{1}{r_2}.$$



Find the conductance of two resistors of 12 ohms and 6 ohms when connected in parallel.

142. What number can be added to 11.7 to obtain $-7\frac{3}{4}$?

143. What number can be multiplied by −0.02 to obtain −625?

R.3 Exponential Notation and Order of Operations

(a) Exponential Notation

Exponential notation is a shorthand device. For $3 \cdot 3 \cdot 3 \cdot 3$, we write 3^4. In the **exponential notation** 3^4, the number 3 is called the **base** and the number 4 is called the **exponent.**

EXPONENTIAL NOTATION

Exponential notation a^n, where n is an integer greater than 1, means

$$\underbrace{a \cdot a \cdot a \cdots a \cdot a}_{n \text{ factors}}.$$

We read "a^n" as "a to the nth power," or simply "a to the nth."
We can read "a^2" as "a-squared" and "a^3" as "a-cubed."

Caution!

a^n does *not* mean to multiply n times a. For example, 3^2 means $3 \cdot 3$, or 9, not $3 \cdot 2$, or 6.

EXAMPLES Write exponential notation.

1. $7 \cdot 7 \cdot 7 = 7^3$
2. $xxxxx = x^5$
3. $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \left(\frac{2}{3}\right)^4$

Do Exercises 1–3.

EXAMPLES Evaluate.

4. $9^2 = 9 \cdot 9 = 81$
5. $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$
6. $\left(\frac{7}{8}\right)^2 = \frac{7}{8} \cdot \frac{7}{8} = \frac{49}{64}$
7. $(0.1)^4 = (0.1)(0.1)(0.1)(0.1) = 0.0001$
8. $(-5)^3 = (-5)(-5)(-5) = -125$
9. $-(5^3) = -(5 \cdot 5 \cdot 5) = -125$
10. $-(10)^4 = -(10 \cdot 10 \cdot 10 \cdot 10) = -10{,}000$
11. $(-10)^4 = (-10)(-10)(-10)(-10) = 10{,}000$

Note that $-(10)^4 \neq (-10)^4$, as shown in Examples 10 and 11. In $-(10)^4$, the sign is *outside* the parentheses; in $(-10)^4$, the sign is *inside* the parentheses.

Do Exercises 4–10.

OBJECTIVES

(a) Rewrite expressions with whole-number exponents, and evaluate exponential expressions.

(b) Rewrite expressions with or without negative integers as exponents.

(c) Simplify expressions using the rules for order of operations.

Write exponential notation.

1. $8 \cdot 8 \cdot 8 \cdot 8$
2. *mmmmmm*
3. $\frac{7}{8} \cdot \frac{7}{8} \cdot \frac{7}{8}$

Evaluate.

4. 3^4
5. $\left(\frac{1}{4}\right)^2$
6. $(-10)^6$
7. $(0.2)^3$
8. $(5.8)^4$
9. -4^4
10. $(-3)^4$

Answers

1. 8^4 2. m^6 3. $\left(\frac{7}{8}\right)^3$ 4. 81 5. $\frac{1}{16}$
6. 1,000,000 7. 0.008 8. 1131.6496
9. −256 10. 81

When an exponent is an integer greater than 1, it tells how many times the base occurs as a factor. What happens when the exponent is 1 or 0? We cannot have the base occurring as a factor 1 time or 0 times because there are no products. Look for a pattern below. Think of dividing by 10 on the right.

On this side, the exponents decrease by 1 at each step. ↓		On this side, we divide by 10 at each step. ↓
	$10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$	
	$10^3 = 10 \cdot 10 \cdot 10 = 1000$	
	$10^2 = 10 \cdot 10 = 100$	
	$10^1 = ?$	
	$10^0 = ?$	

In order for the pattern to continue, 10^1 would have to be 10 and 10^0 would have to be 1. We will *agree* that exponents of 1 and 0 have that meaning.

EXPONENTS OF 0 AND 1

For any number a, we agree that a^1 means a.
For any nonzero number a, we agree that a^0 means 1.

EXAMPLES Rewrite without an exponent.

12. $4^1 = 4$

13. $(-97)^1 = -97$

14. $6^0 = 1$

15. $(-37.4)^0 = 1$

Let's consider a justification for not defining 0^0. By examining the pattern $3^0 = 1$, $2^0 = 1$, and $1^0 = 1$, we might think that 0^0 should be 1. However, by examining the pattern $0^3 = 0$, $0^2 = 0$, and $0^1 = 0$, we might think that 0^0 should be 0. To avoid this confusion, mathematicians agree *not* to define 0^0.

Do Exercises 11–15.

Rewrite without exponents.

11. 8^1

12. $(-31)^1$

13. 3^0

14. $(-7)^0$

15. y^0, where $y \neq 0$

b Negative Integers as Exponents

How shall we define negative integers as exponents? Look for a pattern below. Again, think of dividing by 10 on the right.

On this side, the exponents decrease by 1 at each step. ↓		On this side, we divide by 10 at each step. ↓
	$10^2 = 100$	
	$10^1 = 10$	
	$10^0 = 1$	
	$10^{-1} = ?$	
	$10^{-2} = ?$	

In order for the pattern to continue, 10^{-1} would have to be $\frac{1}{10}$ and 10^{-2} would have to be $\frac{1}{100}$. This leads to the following agreement.

NEGATIVE EXPONENTS

For any real number a that is nonzero and any integer n,

$$a^{-n} = \frac{1}{a^n}.$$

Answers

11. 8 12. −31 13. 1 14. 1 15. 1

EXAMPLES Rewrite using a positive exponent. Evaluate, if possible.

16. $y^{-5} = \frac{1}{y^5}$

17. $\frac{1}{t^{-4}} = t^4$

18. $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{(-2)(-2)(-2)} = \frac{1}{-8} = -\frac{1}{8}$

19. $\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\frac{1}{8}} = 1 \cdot \frac{8}{1} = 8$

20. $\left(\frac{2}{5}\right)^{-2} = \frac{1}{\left(\frac{2}{5}\right)^2} = \frac{1}{\frac{4}{25}} = 1 \cdot \frac{25}{4} = \frac{25}{4}$

The numbers a^n and a^{-n} are reciprocals because

$$a^n \cdot a^{-n} = a^n \cdot \frac{1}{a^n} = \frac{a^n}{a^n} = 1.$$

For example, y^3 and y^{-3} are reciprocals:

$$y^3 \cdot y^{-3} = y^3 \cdot \frac{1}{y^3} = \frac{y^3}{y^3} = 1.$$

Caution!

A negative exponent does *not* necessarily indicate that an answer is negative! For example, 3^{-2} means $1/3^2$, or $1/9$, not -9.

Do Exercises 16–20.

Rewrite using a positive exponent. Evaluate, if possible.

16. m^{-4}

17. $(-4)^{-3}$

18. $\frac{1}{x^{-3}}$

19. $\left(\frac{1}{5}\right)^{-3}$

20. $\left(\frac{3}{4}\right)^{-2}$

EXAMPLES Rewrite using a negative exponent.

21. $\frac{1}{x^2} = x^{-2}$

22. $\frac{1}{(-7)^4} = (-7)^{-4}$

Do Exercises 21 and 22.

Rewrite using a negative exponent.

21. $\frac{1}{a^3}$

22. $\frac{1}{(-5)^4}$

(C) Order of Operations

What does $8 + 2 \cdot 5^3$ mean? If we add 8 and 2 and multiply by 5^3, or 125, we get 1250. If we multiply 2 times 125 and add 8, we get 258. Both results cannot be correct. To avoid such difficulties, we make agreements about which operations should be done first.

RULES FOR ORDER OF OPERATIONS

1. Do all the calculations within grouping symbols, like parentheses, before operations outside.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in order from left to right.
4. Do all additions and subtractions in order from left to right.

Most computers and calculators are programmed using these rules.

Answers

16. $\frac{1}{m^4}$ 17. $-\frac{1}{64}$ 18. x^3 19. 125
20. $\frac{16}{9}$ 21. a^{-3} 22. $(-5)^{-4}$

EXAMPLE 23 Simplify: $-43 \cdot 56 - 17$.

There are no parentheses or powers so we start with the third rule.

$-43 \cdot 56 - 17 = -2408 - 17$	Carrying out all multiplications and divisions in order from left to right
$= -2425$	Carrying out all additions and subtractions in order from left to right

EXAMPLE 24 Simplify: $8 + 2 \cdot 5^3$.

$8 + 2 \cdot 5^3 = 8 + 2 \cdot 125$	Evaluating the exponential expression
$= 8 + 250$	Doing the multiplication
$= 258$	Adding

EXAMPLE 25 Simplify and compare: $(8 - 10)^2$ and $8^2 - 10^2$.

$(8 - 10)^2 = (-2)^2 = 4;$

$8^2 - 10^2 = 64 - 100 = -36$

We see that $(8 - 10)^2$ and $8^2 - 10^2$ are *not* the same.

EXAMPLE 26 Simplify: $3^4 + 62 \cdot 8 - 2(29 + 33 \cdot 4)$.

$3^4 + 62 \cdot 8 - 2(29 + 33 \cdot 4)$	
$= 3^4 + 62 \cdot 8 - 2(29 + 132)$	Carrying out operations inside parentheses first; doing the multiplication
$= 3^4 + 62 \cdot 8 - 2(161)$	Completing the addition inside parentheses
$= 81 + 62 \cdot 8 - 2(161)$	Evaluating the exponential expression
$= 81 + 496 - 2(161)$ $= 81 + 496 - 322$	Doing the multiplication in order from left to right
$= 577 - 322$ $= 255$	Doing all additions and subtractions in order from left to right

Simplify.

23. $43 - 52 \cdot 80$

24. $3^5 \div 3^4 \cdot 3^2$

25. $62 \cdot 8 + 4^3 - (5^2 - 64 \div 4)$

26. Simplify and compare: $(7 - 4)^2$ and $7^2 - 4^2$.

Do Exercises 23–26.

When parentheses occur within parentheses, we can make them different shapes, such as [ ] (also called "brackets") and { } (usually called "braces"). Parentheses, brackets, and braces all have the same meaning. When parentheses occur within parentheses, **computations in the *innermost* ones are to be done first.**

EXAMPLE 27 Simplify: $5 - \{6 - [3 - (7 + 3)]\}$.

$5 - \{6 - [3 - (7 + 3)]\} = 5 - \{6 - [3 - 10]\}$	Adding $7 + 3$
$= 5 - \{6 - [-7]\}$	Subtracting $3 - 10$
$= 5 - 13$	Subtracting $6 - [-7]$
$= -8$	

Answers

23. -4117 24. 27 25. 551 26. 9; 33

EXAMPLE 28 Simplify: $7 - [3(2 - 5) - 4(2 + 3)]$.

$$7 - [3(2 - 5) - 4(2 + 3)] = 7 - [3(-3) - 4(5)]$$ Doing the calculations in the innermost grouping symbols first

$$= 7 - [-9 - 20]$$
$$= 7 - [-29]$$
$$= 36$$

Do Exercises 27 and 28.

Simplify.

27. $6 - \{5 - [2 - (8 + 20)]\}$

28. $5 + \{6 - [2 + (5 - 2)]\}$

In addition to the usual grouping symbols—parentheses, brackets, and braces—a fraction bar and absolute-value signs can act as grouping symbols.

EXAMPLE 29 Calculate: $\dfrac{12|7 - 9| + 8 \cdot 5}{3^2 + 2^3}$.

An equivalent expression with brackets as grouping symbols is

$[12|7 - 9| + 8 \cdot 5] \div [3^2 + 2^3]$.

What this shows, in effect, is that we do the calculations in the numerator and in the denominator separately, and then divide the results:

$$\frac{12|7 - 9| + 8 \cdot 5}{3^2 + 2^3} = \frac{12|-2| + 8 \cdot 5}{9 + 8} = \frac{12(2) + 8 \cdot 5}{17}$$ Subtracting inside the absolute-value signs before taking the absolute value

$$= \frac{24 + 40}{17} = \frac{64}{17}.$$

Do Exercises 29 and 30.

Simplify.

29. $\dfrac{8 \cdot 7 - |6 - 8|}{5^2 + 6^3}$

30. $\dfrac{(8 - 3)^2 + (7 - 10)^2}{3^2 - 2^3}$

Calculator Corner

Order of Operations Computations are usually entered on a graphing calculator in the same way in which we would write them. To calculate $5 + 3 \cdot 4$, for example, we press 5 + 3 × 4 ENTER. The result is 17.

When an expression contains grouping symbols, we enter them using the (and) keys. To calculate $7(11 - 2) - 24$, we press 7 (1 1 − 2) − 2 4 ENTER. The result is 39.

Since a fraction bar acts as a grouping symbol, we must supply parentheses when entering some fraction expressions. To calculate $\dfrac{45 + 135}{2 - 17}$, for example, we enter it as $(45 + 135) \div (2 - 17)$. The result is -12.

5+3*4	17
7(11−2)−24	39
(45+135)/(2−17)	−12

Exercises: Calculate.

1. $48 \div 2 \cdot 3 - 4 \cdot 4$

2. $48 \div (2 \cdot 3 - 4) \cdot 4$

3. $\{(25 \cdot 30) \div [(2 \cdot 16) \div (4 \cdot 2)]\} + 15(45 \div 9)$

4. $\dfrac{17^2 - 311}{16 - 7}$

Answers

27. -25 28. 6 29. $\dfrac{54}{241}$ 30. 34

R.3 Exercise Set

For Extra Help
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a Write exponential notation.

1. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

2. $6 \cdot 6 \cdot 6$

3. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

4. $x \cdot x \cdot x \cdot x$

5. mmm

6. $ttttt$

7. $\frac{7}{12} \cdot \frac{7}{12} \cdot \frac{7}{12} \cdot \frac{7}{12}$

8. $(3.8)(3.8)(3.8)(3.8)(3.8)$

9. $(123.7)(123.7)$

10. $\left(-\frac{4}{5}\right)\left(-\frac{4}{5}\right)\left(-\frac{4}{5}\right)$

Evaluate.

11. 2^7

12. 9^3

13. $(-2)^5$

14. $(-7)^2$

15. $\left(\frac{1}{3}\right)^4$

16. $(0.1)^6$

17. $(-4)^3$

18. $(-3)^4$

19. $(-5.6)^2$

20. $\left(\frac{2}{3}\right)^4$

21. 5^1

22. $(\sqrt{6})^1$

23. 34^0

24. $\left(\frac{5}{2}\right)^1$

25. $(\sqrt{6})^0$

26. $(-4)^0$

27. $\left(\frac{7}{8}\right)^1$

28. $(-87)^0$

b Rewrite using a positive exponent. Evaluate, if possible.

29. $\left(\frac{1}{4}\right)^{-2}$

30. $\left(\frac{1}{5}\right)^{-3}$

31. $\left(\frac{2}{3}\right)^{-3}$

32. $\left(\frac{5}{2}\right)^{-4}$

33. y^{-5}

34. x^{-6}

35. $\frac{1}{a^{-2}}$

36. $\frac{1}{y^{-7}}$

37. $(-11)^{-1}$

38. $(-4)^{-3}$

Rewrite using a negative exponent.

39. $\frac{1}{3^4}$

40. $\frac{1}{9^2}$

41. $\frac{1}{b^3}$

42. $\frac{1}{n^5}$

43. $\frac{1}{(-16)^2}$

44. $\frac{1}{(-8)^6}$

C Simplify.

45. $12 - 4(5 - 1)$

46. $6 - 4(8 - 5)$

47. $9[8 - 7(5 - 2)]$

48. $10[7 - 4(8 - 5)]$

49. $[5(8 - 6) + 12] - [24 - (8 - 4)]$

50. $[9(7 - 4) + 19] - [25 - (7 + 3)]$

51. $[64 \div (-4)] \div (-2)$

52. $[48 \div (-3)] \div \left(-\frac{1}{4}\right)$

53. $19(-22) + 60$

54. $30 \cdot 10 - 18 \cdot 25$

55. $(5 + 7)^2$; $5^2 + 7^2$

56. $(9 - 12)^2$; $9^2 - 12^2$

57. $2^3 + 2^4 - 20 \cdot 30$

58. $7 \cdot 8 - 3^2 - 2^3$

59. $5^3 + 36 \cdot 72 - (18 + 25 \cdot 4)$

60. $4^3 + 20 \cdot 10 + 7^2 - 23$

61. $(13 \cdot 2 - 8 \cdot 4)^2$

62. $(9 \cdot 8 + 3 \cdot 3)^2$

63. $4000 \cdot (1 + 0.12)^3$

64. $5000 \cdot (4 + 1.16)^2$

65. $(20 \cdot 4 + 13 \cdot 8)^2 - (39 \cdot 15)^3$

66. $(43 \cdot 6 - 14 \cdot 7)^3 + (33 \cdot 34)^2$

67. $18 - 2 \cdot 3 - 9$

68. $18 - (2 \cdot 3 - 9)$

69. $(18 - 2 \cdot 3) - 9$

70. $(18 - 2)(3 - 9)$

71. $[24 \div (-3)] \div \left(-\frac{1}{2}\right)$

72. $[(-32) \div (-2)] \div (-2)$

73. $15 \cdot (-24) + 50$

74. $30 \cdot 20 - 15 \cdot 24$

75. $4 \div (8 - 10)^2 + 1$

76. $16 \div (19 - 15)^2 - 7$

77. $6^3 + 25 \cdot 71 - (16 + 25 \cdot 4)$

78. $5^3 + 20 \cdot 40 + 8^2 - 29$

79. $5000 \cdot (1 + 0.16)^3$

80. $4000 \cdot (3 + 1.14)^2$

81. $4 \cdot 5 - 2 \cdot 6 + 4$

82. $8(7 - 3)/4$

83. $4 \cdot (6 + 8)/(4 + 3)$

84. $4^3/8$

85. $[2 \cdot (5 - 3)]^2$

86. $5^3 - 7^2$

87. $8(-7) + 6(-5)$

88. $10(-5) + 1(-1)$

89. $19 - 5(-3) + 3$

90. $14 - 2(-6) + 7$

91. $9 \div (-3) + 16 \div 8$

92. $-32 - 8 \div 4 - (-2)$

93. $7 + 10 - (-10 \div 2)$

94. $(3 - 8)^2$

95. $5^2 - 8^2$

96. $28 - 10^3$

97. $20 + 4^3 \div (-8)$

98. $2 \times 10^3 - 5000$

99. $-7(3^4) + 18$

100. $6[9 - (3 - 4)]$

101. $9[(8 - 11) - 13]$

102. $1000 \div (-100) \div 10$

103. $256 \div (-32) \div (-4)$

104. $\frac{20 - 6^2}{9^2 + 3^2}$

105. $\frac{5^2 - |4^3 - 8|}{9^2 - 2^2 - 1^5}$

106. $\frac{4|6 - 7| - 5 \cdot 4}{6 \cdot 7 - 8|4 - 1|}$

107. $\frac{30(8 - 3) - 4(10 - 3)}{10|2 - 6| - 2(5 + 2)}$

108. $\frac{5^3 - 3^2 + 12 \cdot 5}{-32 \div (-16) \div (-4)}$

Skill Maintenance

Find the absolute value. [R.1d]

109. $\left|-\frac{9}{7}\right|$

110. $|2.3|$

111. $|0|$

112. $|-900|$

Compute. [R.2a, c, d]

113. $23 - 56$

114. $-23 - 56$

115. $-23 - (-56)$

116. $-23 + (-56)$

117. $(-10)(2.3)$

118. $(-10)(-2.3)$

119. $10(-2.3)$

120. $\left(-\frac{2}{3}\right)\left(-\frac{15}{16}\right)$

Synthesis

Simplify.

121. $(-2)^0 - (-2)^3 - (-2)^{-1} + (-2)^4 - (-2)^{-2}$

122. $2(6^1 \cdot 6^{-1} - 6^{-1} \cdot 6^0)$

123. Place parentheses in this statement to make it true: $9 \cdot 5 + 2 - 8 \cdot 3 + 1 = 22$.

The symbol means to use your calculator to work a particular exercise.

124. Find each of the following.

$12345679 \cdot 9 = ?$

$12345679 \cdot 18 = ?$

$12345679 \cdot 27 = ?$

Then look for a pattern and find $12345679 \cdot 36$ without the use of a calculator.

125. Find $(0.2)^{(-0.2)^{-1}}$.

126. Determine which is larger: $(\pi)^{\sqrt{2}}$ or $\left(\sqrt{2}\right)^{\pi}$.

127. Find $(2 + 3)^{-1}$ and $2^{-1} + 3^{-1}$ and determine whether they are equivalent.

R.4

PART 2 MANIPULATIONS

Introduction to Algebraic Expressions

OBJECTIVES

a Translate a phrase to an algebraic expression.

b Evaluate an algebraic expression by substitution.

The study of algebra involves the use of equations to solve problems. Equations are constructed from algebraic expressions. The purpose of Part 2 of this chapter is to provide a review of the types of expressions encountered in algebra and ways in which we can manipulate them.

Algebraic Expressions and Their Use

In arithmetic, you worked with expressions such as

$$91 + 76,\quad 26 - 17,\quad 14 \cdot 35,\quad 7 \div 8,\quad \frac{7}{8},\quad \text{and}\quad 5^2 - 3^2.$$

In algebra, we use these as well as expressions like

$$x + 76,\quad 26 - q,\quad 14 \cdot x,\quad d \div t,\quad \frac{d}{t},\quad \text{and}\quad x^2 - y^2.$$

When a letter is used to stand for various numbers, it is called a **variable**. Let $t =$ the number of hours that a passenger jet has been flying. Then t is a variable, because t changes as the flight continues. If a letter represents one particular number, it is called a **constant**. Let $d =$ the number of hours in a day. Then d is a constant.

An **algebraic expression** consists of variables, numbers, and operation signs, such as $+, -, \cdot, \div$. All the expressions above are examples of algebraic expressions. When an equals sign, $=$, is placed between two expressions, an **equation** is formed.

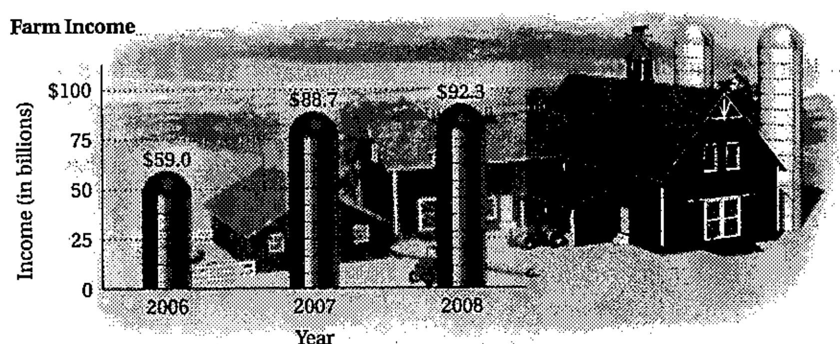
We compare algebraic expressions with equations in the table at left. Note that none of the expressions has an equals sign ($=$).

ALGEBRAIC EXPRESSIONS	EQUATIONS
10	$t = 10$
$x - 5$	$x - 5 = 10$
$11x$	$x - 5 = 11x$
$y^2 + 2y$	$y^2 + 2y = 1 + y$

Do Exercise 1.

1. Which of the following are equations?
   a) $3x + 7$
   b) $-3x - 7 = 18$
   c) $-3(x - 5) + 17$
   d) $7 = t - 4$

Equations can be used to solve applied problems. To illustrate this, consider the bar graph below, which shows farm income for several recent years.



SOURCE: U.S. Department of Agriculture

Answer

1. (b) and (d)

Suppose we want to determine how much higher farm income was in 2008 than in 2007. We can translate this problem to an equation, as follows:

Farm income in 2007	plus	How much more	is	Farm income in 2008
↓	↓	↓	↓	↓
88.7	+	x	=	92.3.

To find the number that x represents, we subtract 88.7 on both sides of the equation:

$$88.7 + x = 92.3$$
$$88.7 + x - 88.7 = 92.3 - 88.7 \quad \text{Subtracting 88.7}$$
$$x = 3.6.$$

We see that farm income was \$3.6 billion higher in 2008 than in 2007.

Do Exercise 2.

2. Refer to the graph on the preceding page. Translate to an equation and solve: How much higher was farm income in 2008 than in 2006?

a Translating to Algebraic Expressions

To translate problems to equations, we need to know that certain words correspond to certain symbols, as shown in the following table.

KEY WORDS

ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION
add	subtract	multiply	divide
sum	difference	product	quotient
plus	minus	times	divided by
total	decreased by	twice	ratio
increased by	less than	of	per
more than			

Expressions like rs represent products and can also be written as $r \cdot s$, $r \times s$, $(r)(s)$, or $r(s)$. The multipliers r and s are also called **factors**. A quotient $m \div 5$ can also be represented as $m/5$ or $\frac{m}{5}$.

EXAMPLE 1 Translate to an algebraic expression: Eight less than some number.

We can use any variable we wish, such as x, y, t, m, n, and so on. Here we let t represent the number. If we knew the number to be 23, then the translation of "eight less than 23" would be $23 - 8$. If we knew the number to be 345, then the translation of "eight less than 345" would be $345 - 8$. Since we are using a variable for the number, the translation is

$t - 8$. *Caution!* $8 - t$ would be incorrect.

Answer

2. $59.0 + x = 92.3$; \$33.3 billion

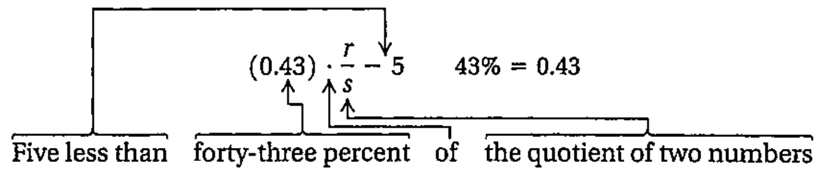
EXAMPLE 2 Translate to an algebraic expression: Twenty-two more than some number.

This time we let y represent the number. If we knew the number to be 47, then the translation would be $47 + 22$, or $22 + 47$. If we knew the number to be 17.95, then the translation would be $17.95 + 22$, or $22 + 17.95$. Since we are using a variable, the translation is

$$y + 22, \quad \text{or} \quad 22 + y.$$

EXAMPLE 3 Translate to an algebraic expression: Five less than forty-three percent of the quotient of two numbers.

We let r and s represent the two numbers.



EXAMPLE 4 Translate each of the following to an algebraic expression.

PHRASE	ALGEBRAIC EXPRESSION
Five *more than* some number	$n + 5$, or $5 + n$
Half *of* a number	$\frac{1}{2}t$, or $\frac{t}{2}$
Five *more than* three *times* some number	$3p + 5$, or $5 + 3p$
The *difference* of two numbers	$x - y$
Six *less than* the *product* of two numbers	$rs - 6$
Seventy-six percent *of* some number	$0.76z$, or $\frac{76}{100}z$
Eight *less than twice* some number	$2x - 8$

Do Exercises 3–8.

(b) Evaluating Algebraic Expressions

When we replace a variable with a number, we say that we are **substituting** for the variable. Carrying out the resulting calculation is called **evaluating the expression.**

EXAMPLE 5 Evaluate $x - y$ when $x = 83$ and $y = 49$.

We substitute 83 for x and 49 for y and carry out the subtraction:

$$x - y = 83 - 49 = 34.$$

The number 34 is called the **value** of the expression.

Translate to an algebraic expression.

3. Sixteen less than some number
4. Forty-seven more than some number
5. Sixteen minus some number
6. One-fourth of some number
7. Six more than eight times some number
8. Eight less than ninety-nine percent of the quotient of two numbers

Answers

3. $x - 16$ 4. $y + 47$, or $47 + y$ 5. $16 - x$
6. $\frac{1}{4}t$ 7. $8x + 6$, or $6 + 8x$
8. $99\% \cdot \frac{a}{b} - 8$, or $(0.99) \cdot \frac{a}{b} - 8$

EXAMPLE 6 Evaluate a/b when $a = -63$ and $b = 7$.

We substitute -63 for a and 7 for b and carry out the division:

$$\frac{a}{b} = \frac{-63}{7} = -9.$$

EXAMPLE 7 Evaluate the expression $3xy + z$ when $x = 2$, $y = -5$, and $z = 7$.

We substitute and carry out the calculations according to the rules for order of operations:

$$3xy + z = 3(2)(-5) + 7 = -30 + 7 = -23.$$

Do Exercises 9–14.

Geometric formulas must often be evaluated in applied problems. In the next example, we use the formula for the area A of a triangle with a base of length b and a height of length h:

$$A = \tfrac{1}{2}bh.$$

EXAMPLE 8 *Area of a Triangular Sail.* The base of a triangular sail is 6.4 m and the height is 8 m. Find the area of the sail.

We substitute 6.4 for b and 8 for h and multiply:

$$A = \tfrac{1}{2}bh = \tfrac{1}{2} \cdot 6.4 \cdot 8$$
$$= 25.6\ \text{m}^2.$$

Do Exercise 15.

EXAMPLE 9 Evaluate $5 + 2(a - 1)^2$ when $a = 4$.

$5 + 2(a - 1)^2 = 5 + 2(4 - 1)^2$	Substituting
$= 5 + 2(3)^2$	Working within parentheses first
$= 5 + 2(9)$	Simplifying 3^2
$= 5 + 18$	Multiplying
$= 23$	Adding

EXAMPLE 10 Evaluate $9 - x^3 + 6 \div 2y^2$ when $x = 2$ and $y = 5$.

$9 - x^3 + 6 \div 2y^2 = 9 - 2^3 + 6 \div 2(5)^2$	Substituting
$= 9 - 8 + 6 \div 2 \cdot 25$	Simplifying 2^3 and 5^2
$= 9 - 8 + 3 \cdot 25$	Dividing
$= 9 - 8 + 75$	Multiplying
$= 1 + 75$	Subtracting
$= 76$	Adding

Do Exercises 16–18.

9. Evaluate $a + b$ when $a = 48$ and $b = 36$.

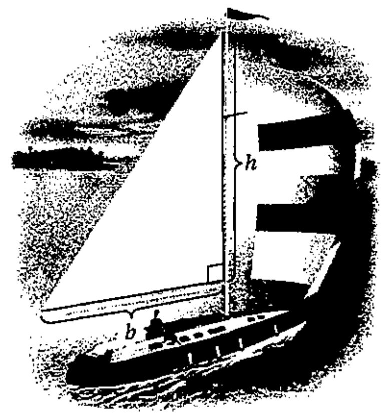
10. Evaluate $x - y$ when $x = -97$ and $y = 29$.

11. Evaluate a/b when $a = 400$ and $b = -8$.

12. Evaluate $8t$ when $t = 15$.

13. Evaluate $4x + 5y$ when $x = -2$ and $y = 10$.

14. Evaluate $7ab - c$ when $a = -3$, $b = 4$, and $c = 62$.



15. Find the area of a triangle when h is 24 ft and b is 8 ft.

16. Evaluate $(x - 3)^2$ when $x = 11$.

17. Evaluate $x^2 - 6x + 9$ when $x = 11$.

18. Evaluate $8 - x^3 + 10 \div 5y^2$ when $x = 4$ and $y = 6$.

Answers

9. 84 10. −126 11. −50
12. 120 13. 42 14. −146
15. 96 ft² 16. 64 17. 64 18. 16

R.4 Exercise Set

For Extra Help
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a Translate each phrase to an algebraic expression.

1. 8 more than b

2. 11 more than t

3. 13.4 less than c

4. 0.203 less than d

5. 5 increased by q

6. 18 increased by z

7. b more than a

8. c more than d

9. x divided by y

10. c divided by h

11. x plus w

12. s added to t

13. m subtracted from n

14. p subtracted from q

15. The sum of p and q

16. The sum of a and b

17. Three times q

18. Twice z

19. -18 multiplied by m

20. The product of -6 and t

21. The product of 17% and your salary

22. 48% of the women attending

23. Megan drove at a speed of 75 mph for t hours on an interstate highway in Arizona. How far did Megan travel?

24. Joe had d dollars before spending \$19.95 on a DVD of the movie *Citizen Kane*. How much did Joe have after the purchase?

25. Jennifer had \$40 before spending x dollars on a pizza. How much remains?

26. Lance drove his pickup truck at a speed of 65 mph for t hours. How far did he travel?

b Evaluate.

27. $23z$, when $z = -4$

28. $57y$, when $y = -8$

29. $\frac{a}{b}$, when $a = -24$ and $b = -8$

30. $\frac{x}{y}$, when $x = 30$ and $y = -6$

31. $\frac{m - n}{8}$, when $m = 36$ and $n = 4$

32. $\frac{5}{p + q}$, when $p = 20$ and $q = 30$

33. $\frac{5z}{y}$, when $z = 9$ and $y = 2$

34. $\frac{18m}{n}$, when $m = 7$ and $n = 18$

35. $2c \div 3b$, when $b = 4$ and $c = 6$

36. $4x - y$, when $x = 3$ and $y = -2$

37. $25 - r^2 + s \div r^2$, when $r = 3$ and $s = 27$

38. $n^3 - 2 + p \div n^2$, when $n = 2$ and $p = 12$

39. $m + n(5 + n^2)$, when $m = 15$ and $n = 3$

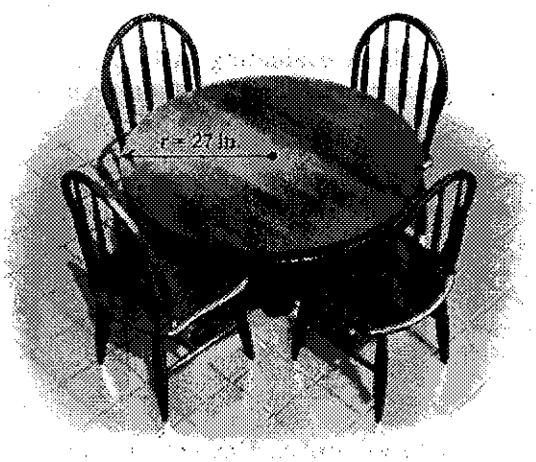
40. $a^2 - 3(a - b)$, when $a = 10$ and $b = -8$

Simple Interest. The **simple interest** I on a principal of P dollars at interest rate r for t years is given by $I = Prt$.

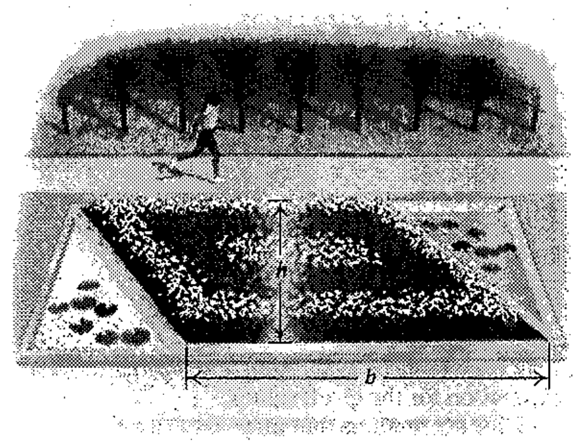
41. Find the simple interest on a principal of $7345 at 6% for 1 year.

42. Find the simple interest on a principal of $18,000 at 4.6% for 2 years. (*Hint*: 4.6% = 0.046.)

43. *Area of a Dining Table.* The area A of a circle with radius r is given by $A = \pi r^2$. The circumference C of the circle is given by $C = 2\pi r$. The radius of Ray and Mary's round oak dining table is 27 in. Find the area and the circumference of the table. Use 3.14 for π.



44. *Area of a Parallelogram.* The area A of a parallelogram with base b and height h is given by $A = bh$. Find the area of a flower garden that is shaped like a parallelogram with a height of 1.9 m and a base of 3.6 m.



Skill Maintenance

Evaluate. [R.3a]

45. 3^5

46. $(-3)^5$

47. $(-10)^2$

48. $(-10)^4$

49. $(-5.3)^2$

50. $\left(\frac{3}{5}\right)^2$

51. $(4.5)^0$

52. $(4.5)^1$

53. $(3x)^1$

54. $(3x)^0$

Synthesis

Translate to an equation.

55. The distance d that a rapid transit train in the Denver airport travels in time t at a speed r is given by speed times time. Write an equation for d.

56. Marlana invests P dollars at 2.7% simple interest. Write an equation for the number of dollars N in the account 1 year from now.

Evaluate.

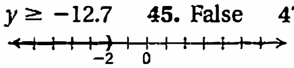
57. $\frac{x + y}{2} + \frac{3y}{2}$, when $x = 2$ and $y = 4$

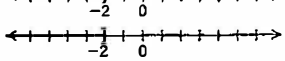
58. $\frac{2.56y}{3.2x}$, when $y = 3$ and $x = 4$

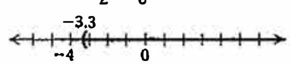
Answers

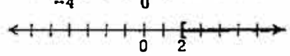
CHAPTER R

Exercise Set R.1, p. 9

1. $1, 12, \sqrt{25}$ **3.** $-6, 0, 1, -\frac{1}{2}, -4, \frac{7}{9}, 12, -\frac{6}{5}, 3.45, 5\frac{1}{2}, \sqrt{25}, -\frac{12}{3}$
5. $-6, 0, 1, -\frac{1}{2}, -4, \frac{7}{9}, 12, -\frac{6}{5}, 3.45, 5\frac{1}{2}, \sqrt{3}, \sqrt{25}, -\frac{12}{3}$, $0.131331333133331\ldots$ **7.** $12, 0$ **9.** $-11, 12, 0$
11. $-\sqrt{5}, \pi, -3.565665666566665\ldots$ **13.** $\{m, a, t, h\}$
15. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ **17.** $\{2, 4, 6, 8, \ldots\}$
19. $\{x \mid x$ is a whole number less than or equal to $5\}$, or $\{x \mid x$ is a whole number less than $6\}$ **21.** $\left\{\frac{a}{b} \middle| a \text{ and } b \text{ are integers and } b \neq 0\right\}$ **23.** $\{x \mid x > -3\}$ **25.** $>$ **27.** $<$ **29.** $<$
31. $<$ **33.** $>$ **35.** $<$ **37.** $>$ **39.** $<$ **41.** $x < -8$
43. $y \geq -12.7$ **45.** False **47.** True
49. 

51. 

53. 

55. 

57. 6 **59.** 28
61. 35 **63.** $\frac{2}{3}$ **65.** 42.8 **67.** 986 **69.** 0 **71.** $\leq$
73. $\leq$ **75.** $\frac{1}{8}\%, 0.3\%, 0.009, 1\%, 1.1\%, \frac{9}{100}, \frac{1}{11}, \frac{99}{1000}, 0.11, \frac{1}{8}, \frac{2}{7}, 0.286$

Exercise Set R.2, p. 19

1. -28 **3.** 5 **5.** -16 **7.** -4 **9.** -10 **11.** -26
13. 1.2 **15.** -8.86 **17.** $-\frac{1}{3}$ **19.** $-\frac{4}{3}$ **21.** $\frac{1}{10}$ **23.** $\frac{7}{20}$
25. 4 **27.** -3.7 **29.** -10 **31.** 0 **33.** -4 **35.** -14
37. 0 **39.** -46 **41.** 5 **43.** 15 **45.** -11.6 **47.** -29.25
49. $-\frac{7}{2}$ **51.** $-\frac{1}{4}$ **53.** $-\frac{19}{12}$ **55.** $-\frac{7}{15}$ **57.** -21 **59.** -8
61. 24 **63.** -112 **65.** 34.2 **67.** $-\frac{12}{35}$ **69.** 2 **71.** 60
73. 26.46 **75.** 1 **77.** $-\frac{8}{27}$ **79.** -2 **81.** -7 **83.** 7
85. 0.3 **87.** Not defined **89.** 0 **91.** Not defined **93.** $\frac{4}{3}$
95. $-\frac{8}{7}$ **97.** $\frac{1}{25}$ **99.** 5 **101.** $-\frac{b}{a}$ **103.** $-\frac{6}{77}$ **105.** 25
107. -6 **109.** 5 **111.** -120 **113.** $-\frac{9}{8}$ **115.** $\frac{5}{3}$ **117.** $\frac{3}{2}$
119. $\frac{9}{64}$ **121.** -2 **123.** $\frac{12}{13}$, or $0.\overline{923076}$ **125.** $-\frac{81}{50}$, or -1.62
127. Not defined **129.** $-\frac{2}{3}, \frac{3}{2}; \frac{5}{4}, -\frac{4}{5}$; 0, does not exist; $-1, 1$; $4.5, -\frac{1}{4.5}; -x, \frac{1}{x}$ **131.** $26, 0$ **132.** 26 **133.** $-13, 26, 0$
134. $\sqrt{3}, \pi, 4.57557555755557\ldots$ **135.** $-12.47, -13, 26, 0, -\frac{23}{32}, \frac{7}{11}$ **136.** $\sqrt{3}, -12.47, -13, 26, \pi, 0, -\frac{23}{32}, \frac{7}{11}$, $4.57557555755557\ldots$ **137.** $<$ **138.** $>$ **139.** $<$ **140.** $>$
141. $\frac{1}{4}$ **143.** 31,250

Calculator Corner, p. 27

1. 56 **2.** 96 **3.** 262.5 **4.** $-2.\overline{4}$, or $-\frac{22}{9}$

Exercise Set R.3, p. 28

1. 4^5 **3.** 5^6 **5.** m^3 **7.** $\left(\frac{7}{12}\right)^4$ **9.** $(123.7)^2$ **11.** 128
13. -32 **15.** $\frac{1}{81}$ **17.** -64 **19.** 31.36 **21.** 5 **23.** 1
25. 1 **27.** $\frac{7}{8}$ **29.** 16 **31.** $\frac{27}{8}$ **33.** $\frac{1}{y^5}$ **35.** a^2 **37.** $-\frac{1}{11}$
39. 3^{-4} **41.** b^{-3} **43.** $(-16)^{-2}$ **45.** -4 **47.** -117
49. 2 **51.** 8 **53.** -358 **55.** 144; 74 **57.** -576
59. 2599 **61.** 36 **63.** 5619.712 **65.** $-200{,}167{,}769$
67. 3 **69.** 3 **71.** 16 **73.** -310 **75.** 2 **77.** 1875
79. 7804.48 **81.** 12 **83.** 8 **85.** 16 **87.** -86 **89.** 37
91. -1 **93.** 22 **95.** -39 **97.** 12 **99.** -549 **101.** -144
103. 2 **105.** $-\frac{31}{76}$ **107.** $\frac{61}{13}$ **109.** $\frac{9}{7}$ **110.** 2.3 **111.** 0
112. 900 **113.** -33 **114.** -79 **115.** 33 **116.** -79
117. -23 **118.** 23 **119.** -23 **120.** $\frac{5}{8}$ **121.** $25\frac{1}{4}$
123. $9 \cdot 5 + 2 - (8 \cdot 3 + 1) = 22$ **125.** 3125
127. $(2 + 3)^{-1} = (5)^{-1} = \frac{1}{5}$; $2^{-1} + 3^{-1} = \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$; so $(2 + 3)^{-1} \neq 2^{-1} + 3^{-1}$.

Exercise Set R.4, p. 36

1. $b + 8$, or $8 + b$ **3.** $c - 13.4$ **5.** $5 + q$, or $q + 5$
7. $a + b$, or $b + a$ **9.** $x \div y$, or $\frac{x}{y}$ **11.** $x + w$, or $w + x$
13. $n - m$ **15.** $p + q$, or $q + p$ **17.** $3q$ **19.** $-18m$
21. $17\%s$, or $0.17s$ **23.** $75t$ **25.** $\$40 - x$ **27.** -92
29. 3 **31.** 4 **33.** $\frac{45}{2}$, or 22.5 **35.** 16 **37.** 19 **39.** 57
41. \$440.70 **43.** $A = 2289.06\text{ in}^2$; $C = 169.56$ in. **45.** 243
46. -243 **47.** 100 **48.** 10,000 **49.** 28.09 **50.** $\frac{9}{25}$
51. 1 **52.** 4.5 **53.** $3x$ **54.** 1 **55.** $d = r \cdot t$ **57.** 9

DELAWARE TECHNICAL COMMUNITY COLLEGE

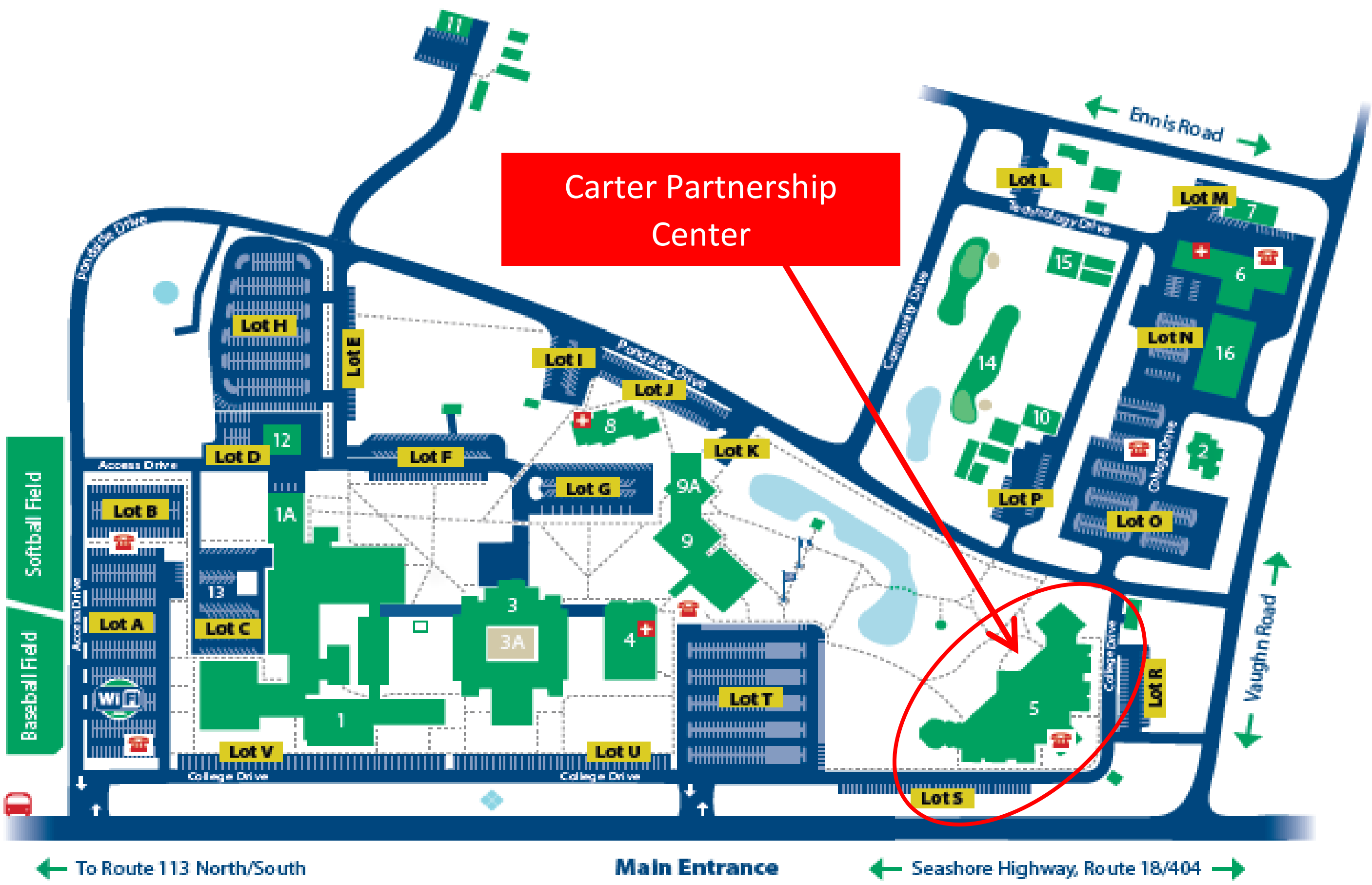
GUIDE TO GETTING AROUND THE JACK F. OWENS CAMPUS

LOCATION	KEY	ROOMS
• Jason Technology Center	1	001-199
• Campus Store	1A	
• Energy House	2	200 Series
• Arts & Science Center	3	300 Series
• Arts & Science Center Courtyard	3A	
• Stephen J. Betze Library	4	400 Series
• Carter Partnership Center	5	500 Series
• Trade & Industry Building	6	600 Series
• Environmental Training Center	7	700 Series
• Child Development Center	8	800 Series

LOCATION	KEY	ROOMS
• Student Services Center	9	900 Series
• Campus Dining	9A	
• Agriculture Education Building	10	1100 Series
• Production Agriculture Education Building	11	1700 Series
• Facilities/Shipping & Receiving Building	12	
• Employee Parking	13	
• Turf Grass Lab	14	
• Environmental Training Lab	15	
• Automotive Center of Excellence	16	

SYMBOL KEY

- Walkways
- Covered Walk
- Bridge
- Lot Parking Lot Identifier
- Public Safety Information
- First Aid Kit
- Dart Bus Stop
- WiFi Parking Lot WiFi Area



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